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THE ELEMENTS

OF

PRACTICAL HYDRAULICS:

FOR THE USE OF

STUDENTS IN ENGINEERING.

BY

SAMUEL DOWNING, LL.D.,

PROFESSOR OF CIVIL ENGINEERING IN THE UNIVERSITY OF DUBLIN.

Second Edition, Revised and Enlarged.

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PREFACE.

The first edition of this work having been favourably received, the Author has very carefully revised the present, and added many Plates and Woodcuts, with several new Examples, of a character, it is hoped, adapted to train the mind of a student to the consideration of practical subjects in a practical point of view.

The work is still, what it at the first was intended to be, a Text-book for students in the Engineering School of this University; as such, it deals with first principles, and their applications to practical construction, rather than giving a collection of Hydraulic Tables, or other matter chiefly useful to those engaged in the actual construction of Hydraulic works. The profession is now gradually assuming the form of separate and distinct specialities,—not the least interesting or important of which is Hydraulic Engineering; but in a well-arranged course of instruction for a University School of Engineering, the principles of all these branches of the profession should be taught to all the students, whatever their destination may be. It was to meet the want of a Text-book of this character, for Hydraulic Engineering, that this work was undertaken.

The admirable experiments carried on at the Lowell Mills, Massachusets, U. S., by J. B. Francis, and published by him, have been quoted in this edition; and the results, as to the discharge over weirs, have been compared with former experimenters, and exhibited in the form of diagrams.

The Author desires to acknowledge, with many thanks, the liberality of the Board in assisting in the expense of this edition, contrary to their rule as to second editions; but this cannot be drawn into a precedent, as it was an acknowledgment, on their part, of the successful drainage of the College Park by the Author.

In all cases of quotation, the names of the authors, whose works were made use of, have been fully given.

To those who would wish to pursue the subject still further, the following works are recommended:—

Beardmore's Hydraulic Tables, 2nd edition.
Neville's Hydraulic Formulæ and Tables, 2nd edition.
Bennett's translation of D'Aubuisson.
J. B. Francis, Lowell Experiments.
Morin's Hydraulique, 2nd edition.

PREFACE TO THE FIRST EDITION.

THE following work is intended as a Text-book for the Students in the School of Engineering, Trinity College, Dublin, in that branch of their intended profession which has reference to Practical Hydraulics.

The first two chapters are in a great degree a translation of D'Aubuisson's well-known work, "Traitè d'Hydraulique a l'usage des Ingenieurs," which has obtained extensive circulation on the Continent, as well as in England and America. The third chapter, relating to the flow of water in artificial channels, rivers, and pipes, is founded on the formula for the uniform motion of water, which is in very general use amongst English engineers, and possesses great advantages over that given by D'Aubuisson, in simplicity and facility of application. Indeed, we do not find that this more complex formula has obtained the full confidence of foreign engineers themselves. Thus M. Girard, who constructed the navigable channel intended to supply the street fountains of Paris with water derived from the river l'Ourcq—being in possession of all the necessary dimensions and data, except the rate of inclination of the surface in the longitudinal section—calculated this last by that more complicated formula of which we speak, but finally decided upon constructing the line with a fall nearly double that thus obtained (§ 120). The formula given in the third chapter has, moreover, served to determine the proportions and dimensions of many English works of the greatest magnitude, which have, when completed, been found to fulfil perfectly their intended objects; and consequently it may be considered as based on an experience the most reliable, and, of all others, the most valuable to the civil engineer.

Throughout this work, the only unit made use of is the foot and the cubic foot. We have in English works on Hydraulics a great variety of units: the gallon, the cubic foot, the ton, the cubic yard, the hogshead; and for length, the yard, foot, and inch, which, with the absence of decimal subdivisions in our weights and measures, is always perplexing to the reader.

As soon as the student has become familiar with the value of the inches in a foot expressed decimally, it is hoped that this arrangement will be found useful. Of the eleven decimal fractions for the inches in a foot, five are well known, namely, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and the rest may be readily remembered. It will be observed, also, that the eighth of an inch is very nearly 0.01 ft.

Inches.	Foot.	Inches.	Foot.
1	$\frac{1}{12}$, 0.0833	7	$\frac{7}{12}$, 0.5833
2	$\frac{1}{6}$, 0.1666	8	$\frac{2}{3}$, 0.6666
3	$\frac{1}{4}$, 0.2500	9	$\frac{3}{4}$, 0.7500
4	$\frac{1}{3}$, 0.3333	10	$\frac{5}{6}$, 0.8333
5	$\frac{5}{12}$, 0.4166	11	$\frac{11}{12}$, 0.9166
6	$\frac{1}{2}$, 0.5000	12	$\frac{12}{12}$, 1.0000

The Author takes this opportunity of returning thanks to the Board for the liberality they have shown in defraying the greater part of the expenses of this Work.

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INTRODUCTION.

1. The science of Hydraulics has for its object the knowledge of the phenomena of fluids in motion, and of the laws which regulate the production of these phenomena.

Applied as an art, its object is to render this knowledge available in the designs of the civil engineer, as in the determination of the dimensions of pipes for conveying water, gas, or air, and also in works for the collecting, conveying, and distributing the necessary supply of water, for mill-power, or for the summit-levels of canals ; or for the supply of cities; and, generally, of all such works as depend fòr their suitable construction and proportions upon the result of calculations requiring a knowledge of the pressure and motion of fluids.

2. Fluids are defined to be bodies whose particles, by reason of their extreme mobility, yield to every the least force ; they have, however, a certain degree of adherence or viscidity which binds them together. These bodies are divided into two classes, the incompressible, or fluids properly so called, and the elastic. Water is the more common type of the former, and the atmosphere of the latter.

3. Although all fluids, as indeed every substance in nature, is in strictness both compressible and elastic, yet the difference in degree is so marked, and the distinction in the expression of their laws so essential under this point of view, that this division must be retained, and hence we have Hydraulics, properly so called, treating of the incompressible fluids, especially water, and Aerometry, treating of the atmosphere and gases.

4. Before entering on the former we must state the values of two quantities which occur in all calculations in Hydraulics, namely, the weight of water, and the measure of the force of gravity.

In statements of this nature we should carefully distinguish between those that are mere arbitrary definitions and those which are but inferences from the definition, and also those which are natural quantities, determined by experiments.

A cubic foot of water weighs 62.32106 lbs. avoirdupois, the foot being the third part of a brass bar, constructed by Mr. Bird about the year 1760 from the mean of several old and inconsistent standards then existing, and which bar was in 1824 declared by Act of Parliament to be the unit of measures of length, and named the "Imperial Standard Yard."

The pound avoirdupois was by the same Act defined to be 7000 grs., the grain being thus determined:—A weight of two pounds Troy having been constructed by the same artist that made the standard yard above mentioned, the half of it (one pound Troy) was divided into 5760 grs. (12 oz. × 20 dwts. × 24 grs.), and 7000 of these grains are equal to one pound avoirdupois.

Measures of capacity have for their unit the gallon, which was defined to contain 10 lbs. av. weight of water,—the thermometer at 62° Fahr., and the barometer standing at 30 inches.

A cubic inch of water, at the same temperature and pressure, was found by experiment to weigh in air 252.458 grs.; and hence, if we divide 70000 by 252.458, we shall have the number of cubic inches in the imperial gallon, which, therefore, is equal to 277.274 cubic inches; and if we multiply 252.458 by 1728, the number of cubic inches in one cubic foot, we have, as above, 62.32106 lbs.: hence, also, 6.23 imperial gallons to one cubic foot.

The cubic foot of water is, in practice, taken approximately, as equal to 62.5 lbs. av., or 1000 ozs. at 16 ozs. to the lb.; giving 36 cubic feet to a ton, or 6 tons to a cubic fathom.

The measure of the force of gravity is the velocity acquired in one second by a body falling freely from a state of rest, and is equal to 32.1948 feet per second, and always denoted by the letter g.

5. The Mètre adopted in France in 1798 as the unit of lineal measures is equal to 39.37079 inches. It is therefore 3.28089 feet, and is subdivided decimally into the decimetre, the centimetre, and the millimètre; and in order to assist the forming a clear idea of the comparative value of the French and English lineal measures, they have been drawn up to 1 ft. in length, on Plate I., from a common zero. The cubic decimetre of water at 39.38° F., and barometer at 29.922 inches, is the standard of weight named the Kilogramme, = 2.20485 lbs. av.; hence 1000 kilos = 1 ton very nearly.

ELEMENTS OF HYDRAULICS.

CHAPTER I.

ON THE FLOW OF WATER THROUGH AN ORIFICE CONTAINED IN A VESSEL.

6. THE vessel from whence the water issues may be, first, maintained at a constant height of surface; or, secondly, it may receive no supply, and, consequently, be exhausted; or, thirdly, the orifice, instead of discharging freely into the air, may do so into another reservoir, under more or less resisting counter-pressure: and hence *three divisions* of this part of the subject. The second division also includes the cases in which the level of the surface gradually rises or falls, from the supply being greater or less than the discharge through the orifice.

7. The opening through which the water flows may be placed either in the bottom or in one of the sides of the experimental tank, most generally the latter, in which case the surface of the water in the basin should be above the upper edge of the orifice: this orifice is opened either in a thin plate, —that is to say, in a plate whose thickness is at the most less than half the diameter of the orifice, if circular, or smallest dimension, if rectangular;—or else it is furnished with an adjutage, or short tube, sometimes cylindrical, sometimes conical, converging towards an external point, less often diverging. An orifice placed in a very thick plate would evidently be equivalent to one of the same diameter if placed in a thin plate, with an adjutage attached.

We may also have the surface of the fluid below the upper edge of the orifice; that part of the border or circumference is then as if it did not exist, and very frequently it is not applied, and the opening is then unlimited on its upper part, and is called an overfall or weir. The laws of the flow of water in this second case offer some peculiarities, and form the subject of a separate investigation. When the surface reaches but up to a very small height above the opening, we also have special circumstances: this case is intermediate between the two others first mentioned.

Before entering upon them it is necessary to state briefly the general principles of the flow of water, and the modifications which the "contraction" of the fluid vein suffers in passing through the various orifices to be noticed. The vertical distance of the surface of the fluid above the centre of gravity of the orifice is called the *charge* of the water upon the orifice, or the *head* under which the flow takes place. This point is not the true depth at which the mean velocity is found, but may without any sensible error be taken to represent it; the exact determination of it will be found in §§ 44 to 48.

8. *Velocity of Water flowing from an Orifice.*—Let a vessel X (Fig. 1), maintained constantly full of water up to the level AB, have upon the horizontal faces CD and EF the orifices M and N; the fluid will issue in vertical jets, which will rise almost to the level of the water AK; they would rise fully up to it but for the resistance of the air. Now, by the first principles of Dynamics, in order that a body impelled in a vertical direction should reach to any height, it is necessary that at the point of departure it should have had a velocity equal to that which it would have acquired in falling freely from that height; consequently, the particles of the fluid must have had a velocity nearly equal to that due to the charge—that is, to the height of the surface of the water above the orifices,—the only supposition in the application of the principle to the flow of water being that the particles of the fluid are perfectly independent of each other after they leave the orifice.

So also, if upon a vertical face BR an orifice O be placed, we shall see below that from the respective values of the lines OP and PQ, the fluid must have issued from O with a velocity due to the height OB. It would issue with a velocity due to BR, if the orifice had been opened in the bottom RT of the vessel; and the velocity is the same in O, in O_1, and O_2, the directions being different, but the charge the same. This truth holds good for different orifices, whatever may be the ratio of the area of the orifice to the horizontal section of the water in the vessel, provided that the level of the water is kept constantly the same, and tranquil; which last, however, cannot be attained, if the orifice be large in proportion to that surface —the water of the supply, in that case, producing disturbing movements in the reservoir.

A second method of determining the velocity is by measuring the curve of the path of a jet of water issuing from an orifice in the vertical side of a cistern. To have a clear notion of this method, it is necessary to recall the following principles:—When a body is projected in any direction AY (Fig. 13) with a certain velocity, the combined action of this velocity with the force of gravity causes it to describe a curved path, AMB. If the velocity, and consequently the resistance of the air, be not very great, the curve is a parabola. The demonstration of this may well be repeated here, though given in many works.

Let v (Fig. 13) be the velocity with which the body is sent forth in the direction of AY, and t the time spent in reaching the point N; then, since the velocity in the direction AN is uniform, AN = vt; on the other hand, if the body had been only under the action of the accelerating force of gravity, it would have descended from A to a point P during that same interval t, such that we should have AP = $\frac{1}{2}gt^2$. If we complete the parallelogram APMN, the point M will have been reached under the joint action of these movements in the same time t in which the point P was attained under the sole accelerating force; and it will have, therefore, traversed the arc of the curve, whose abscissa will be AP, and ordinate MP, parallel to the axis AY. Let x = AP and y = MP, we have therefore

$$(a) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad x = \frac{gt^2}{2} \text{ and}$$

$$(b) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad y = vt.$$

From (b) we have $t = \frac{y}{v}$ and $t^2 = \frac{y^2}{v^2}$; substituting this value of t^2 in (a), we have $\frac{gy^2}{2v^2} = x$, or $y^2 = \frac{2v^2x}{g} = \frac{4v^2x}{2g}$, or calling h the height due to the velocity v, and remembering that $\frac{v^2}{2g} = h$, we have

$$(c) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad y^2 = 4hx,$$

which is the well-known equation of the parabola, of which $4h$ is the parameter.

Hence this theorem, "that a heavy body projected with any force whatsoever describes a parabola, whose parameter is equal to four times the height due to the velocity of projection."

This truth, which has been proved for any body in general, holds good also for a jet of water issuing from an orifice (Fig. 14). If this orifice be opened in a vertical plate, the axis of projection being horizontal, the ordinates will be horizontal,—that is, the distances of the different points of the jet from the vertical let down from the centre of the orifice; and if through any point C of this vertical we draw a horizontal plane, then, according to the theorem, the square of the distance CD—called the range of the jet—taken in this plane (or generally of any distance MP), divided by four times the corresponding fall, AP, will give the height due to the velocity of exit, as from (*c*) we have $h = \frac{y^2}{4x}$.

And the permanent form that the jet of water assumes being identical with the path of any single particle acted on by the same forces, we are enabled to use it as a mode of measuring the velocity of the water at its issue from the orifice. A vertical rod divided into any scale of equal parts, and firmly fixed, having its zero at the centre of the orifice, has applied to it at right angles another rod similarly divided, and having a stock like a T square, so as to slide up or down the vertical fixed rod, and its zero being in the vertical let fall from the mouth of the orifice, we can then measure any ordinate PM and corresponding abscissa AP. Hence, by measuring y and x, we can calculate h, the height due to the velocity of exit, from the formula just given. Dividing both sides of equation (*c*) by $4x$ we have $h = \frac{y^2}{4x}$, and comparing h so found with H, the charge, we find them very nearly coincident, as in the following Table of experiments by Bossut:—

Curve of the Jet.		Height due to Velocity of Exit.	Charge.	Differences.
Abscissa = x	Ordinate = y			
Feet.	Feet.	Feet.	Feet.	Feet.
20.598	24.698	7.404	7.511	0.107
15.284	27.716	12.564	12.890	0.326
4.624	20.500	22.720	23.583	0.863

Thus $\frac{24.698^2}{4 \times 20.598} = \frac{610.1}{82.4} = 7.404$, and so of the other numbers in the third column.

The difference between the third and fourth columns increases with the charge, and we should expect it to be so, since the cause of this difference—the resistance of the air—increases as the square of the velocity, and, consequently, nearly as the charge. Had it not been for this, the difference had been very nearly equal to zero, and the velocity at the section of contraction, as mentioned in p. 4, § 8, is truly stated as equal to that due to the charge. This general proposition may consequently be laid down:—"Water flowing through an orifice in a vertical thin plate issues with a velocity, *q. p.*, equal to that due to the charge."

Thus the velocity acquired by a body falling freely by the force of gravitation from the height H, is equal to that of the fluid as it issues from the orifice; that is—

$$V = \sqrt{2g\mathrm{H}},$$

called after Toricelli, its discoverer, the Torricellian theorem, in which H is the "charge," measured from the surface down to the centre of gravity of the orifice, and g, the dynamical measure of the force of gravitation, being the rate or number of feet per second, with which a body falling freely is moving at the end of the first second.

9. The following table exhibits the results of experiments by Castel, D'Aubuisson, Bossut, Poncelet, &c., also proving this truth.

It will be observed that the charges vary from 1 to 200 and more, and the sections of the orifices from 1 to 500, and yet in all cases the velocities have followed the ratio of the square roots of the charges, minute discrepancies, sometimes giving too great a number, and sometimes too small, being inseparable from experiments of this nature.

The actual object of measurement in the experiments was the *quantity* discharged in a given time; but it is evident that, with the *same orifice*, the discharge is exactly proportional to the velocity with which the fluid issues, and, therefore, that column in the Table which expresses the gauged discharges, reference being made to some one discharge as a unit, also expresses the velocities.

TABLE *showing that the Velocities are proportional to the square roots of the Charges.*

Diameter of the Orifice.	Charge above the Orifice.	Series of	
		Square Roots of the Charges.	Discharges or Velocities.
Feet.	Feet.		
0.0328	0.085	1.000	1.000
	0.098	1.074	1.064
	0.131	1.241	1.244
	0.164	1.386	1.393
	0.196	1.519	1.524
0.088	4.265	1.000	1.000
	9.580	1.500	1.497
	12.500	1.713	1.707
0.265	7.677	1.000	1.000
	12.500	1.305	1.301
	22.179	1.738	1.692
0.531	6.922	1.000	1.000
	12.008	1.316	1.315
Square Orifice 0.656 by 0.656	1.312	1.000	1.000
	2.296	1.323	1.330
	3.281	1.581	1.590
	4.265	1.803	1.806
	5.249	2.000	2.000

10. The general principle, that the velocities are as the square roots of the charges, as also the theorem of Torricelli (§ 8) in the case in which it is applicable, extends to every kind of fluids,—to mercury, oils, alcohols, and even aeriform fluids; so that the velocity with which each of them issues from an orifice is independent of its particular nature, and of its density, it depends solely on the charge. Experiment demonstrates this, and very simple reasoning suffices to show its truth. Take the case of mercury: the particles situated immediately in front of the orifice, and in which it is necessary to create a certain velocity, are, it is true, fourteen times more dense than those of water, and they, consequently, oppose to motion a resistance fourteen times greater than it would do; but the mass also which presses upon these particles, and produces the velocity of exit, being greater in the

same proportion, gives a motive force fourteen times greater. Thus a compensation exists, and the velocity impressed remains the same; and, in like manner, it may be proved for a fluid lighter than water.

11. The proposition that has now been laid down with respect to the velocity of water issuing through an orifice is equally true in cases when the discharge takes place *in vacuo*, the velocity is always the same, with the same head, whatever be the pressure upon the free surface of the water in the vessel, provided the jet of water at its exit from the orifice be subject to an equal exterior pressure. But the velocity will be very different from that due to H, if the pressures be not equal upon these two surfaces.

If the pressure per square inch be greater against the orifice at A than upon the free surface of the water BC (Fig. 2), then the excess of the former above that on the free surface must be less than that of a column of the fluid whose height is the vertical distance of the orifice A, below the surface BC, otherwise there could be no discharge.

Let us, then, take an horizontal plane DE, below the plane BC, and at such distance from it that the weight of a column of the water contained between the two planes, and whose base is the unit of surface, may be equal to the excess of pressure at A, of which we are speaking,—the water in the vessel having but a very slight degree of motion, on account of the relatively small area of the orifice, which is always understood to subsist; and, therefore, we may assume the pressures to be transmitted, as if the contained liquid was in equilibrium. The pressure, then, which exists upon any point in the plane DE will be equal to that upon any point in BC, *plus* the supposed excess of pressure against the exterior of the orifice; and, therefore, the pressure will be the same upon the plane DE, and that against the orifice at A. The liquid below the plane DE is then in the same condition as if that contained between BC and DE were removed, and the free surface and exterior of the orifice were under equal pressures; and thus the same formula will represent the velocity:

$$V = \sqrt{2g\,(\mathrm{H} - h_1)},$$

h_1 denoting the depth of the plane DE below BC.

12. If the exterior pressure on A were less than that upon the surface BC, we may conceive the excess of pressure upon it to be produced by a liquid of the same specific gravity as

that in the vessel, applied above BC and terminating in a free surface D′E′, situated at such height that the vertical distance represents, as before, the column of the liquid whose pressure is equal to the excess of the pressure on BC above that against the orifice at A. The flow, then, will take place with the same velocity as if the free surface of the liquid, instead of being in the plane BC, and supporting this excess of pressure, were at D′E′, and supported the same pressure as the orifice at A; the formula will then be—

$$V = \sqrt{2g(\mathrm{H} + h_2)},$$

if we take h_2 equal to the vertical distance of D′E′ above BC. We see thus that a diminution or augmentation of the pressure upon the free surface of the liquid in the vessel, without any change in that against the orifice at A, causes a corresponding diminution or augmentation in the velocity of the issuing fluid, and, on the contrary, that a diminution or augmentation of the pressure against the orifice without any change in that upon the free surface, causes a corresponding augmentation or diminution in this velocity.

EXAMPLES.—The condenser of a low-pressure steam-engine offers an example of the second case; for, let us suppose a vacuum of 25 inches of mercury to be maintained, and that the head of water in the cistern (Fig. 3) supplying the jet of cold water which effects the condensation, were 2 feet above the point at which it enters this partial vacuum, then the actual head producing the flow is $2 + 28.25 = 30.25$ feet, for, pure mercury being 13.56 times heavier than water, we have the height of a column of water which would balance that of 25 inches of mercury equal to $25 \times 13.56 = 339$ inches, or 28.25 feet.

The self-acting contrivance for supplying the feed-water to low-pressure boilers (Fig. 4) comes under the first case: the pressure being supposed 5 lbs. per inch above the atmosphere, it is required to place the cistern of the supply so high, that on the opening of the valve *a*, by the float *b* descending below the proper level, the water may enter against the pressure of the steam. Now, as the cubic foot of water weighs 62.5 lbs., a column 1 foot high and 1 square inch base weighs $\frac{6.25}{144} = 0.434$ lbs., and, therefore, the height of the column of water to balance any given pressure expressed in pounds per square inch is found by dividing that number by 0.434 in this case, $5 \div 0.434 = 11.52$ feet: this gives exact equilibrium; the additional head in order that it may enter with due rapidity (from 2 to 4 feet generally) will depend upon the consumption of the boiler and the area of the supply pipe. It is evident that this mode of supply is not convenient in high-pressure boilers; for suppose the pressure to be 50 lbs. per inch, then the height to produce equilibrium will be 115.2 feet. The pressure in a hydraulic press is frequently 3 tons per square inch, equal to (3×2240) 6720 lbs., and $6720 \div 0.434 = 15484$ feet.

If, instead of a free surface in the cistern, we had supposed a solid piston or plunger to press on the enclosed water, the head should in like manner be calculated, by turning the pressure per square inch on the piston into vertical feet of water.

13. Having thus established the law of the velocity of a fluid issuing from an orifice, let us proceed to apply it to the determination of its discharge, which is defined to be the volume of the fluid which escapes in the unit of time, i. e. one second.

If the mean velocity of all the particles was that due to the "charge" H, then this velocity, which is called the theoretic velocity, would be $\sqrt{2gH}$; and if at the same time the particles issued from all points of the orifice in parallel threads, it is evident that the volume of water flowing out in a second would be equal to the volume of a prism which would have the orifice for its base, and that velocity for its length; and, calling S the area or section of the orifice, the volume of water, or of the prism, would be—

$$S \times V = S \sqrt{2gH}.$$

This is the theoretic discharge.

14. But the actual discharge is always less than this. In order to have an exact idea of the phenomena, let us consider the fluid vein a short distance after its issue from the orifice, and let us suppose it cut by a plane perpendicular to its direction. It is manifest that the discharge will be equal to the product of the section by the mean velocity of all the several threads at the moment they intersect the plane of the section. If this section was equal to that of the orifice, and if this velocity was that due to the charge, then the actual discharge would also be equal to the theoretic discharge. But whether from the section of the vein being considerably less than that of the orifice,—as in the flow through orifices in a thin plate, or from the velocity being considerably less than that due to the charge, as in cylindrical adjutages; or, again, from a di-

minution in both the section and the velocity, as in certain conical adjutages,—it always results, that the actual discharge is in every case less than the theoretic, and, in order to reduce this last to the former, it is necessary to multiply it by some fraction. Let m represent this fraction, and Q the actual discharge in one second, we shall then have—

$$Q = m\,S\,\sqrt{2g.\mathrm{H}}.$$

And designating the volume of water flowing off in the time T seconds by Q' we shall have—

$$Q' = m\,S\,T\sqrt{2g\mathrm{H}}.$$

Whether the diminution in the discharge arises from a diminution in the section, or in the velocity, it is always a consequence of the contraction which the fluid vein suffers in passing through the orifice, and thus the multiplier m, or "coefficient for the reduction of the theoretic to the actual discharge," is generally called the "*coefficient of contraction*," and is taken to represent the aggregate effect of all circumstances tending to diminish the discharge. Its accurate determination is of the greatest importance; upon its degree of exactness depends that of the results we obtain when we would apply to practice formulæ upon the flow of water. We shall now proceed to give the results of experiments on the value of the symbol m, making some preliminary statements upon the cause of the "contraction," and the nature of its effects; and also upon the form of the fluid vein—the orifice being circular—its dimensions, and the effect of the form upon the discharge.

15. *Cause of the Contraction.*—If we take a glass vessel (Fig. 5), in the side of which is an orifice through which the water flows, and render visible the movement of the molecules of the water in the vessel by disseminating through it a substance of equal specific gravity, and very minute, or by producing within the water some light chemical precipitation, such as occurs when we let fall a few drops of nitrate of silver in water slightly saline,—we then see at a small distance from the orifice, as, for instance, about an inch, when its diameter is three-eighths of an inch—the fluid molecules converge from all parts towards the orifice, describing curved lines, and, finally, as if approaching a centre of attraction, issue forth with a rapidly increased motion.

The convergence of the directions that they had within the vessel at the moment of their arrival at the orifice still continues for a short distance after they have passed out, so that we can plainly see the fluid vein gradually diminish, and become contracted up to the place where the particles, from the effect of their mutual action, and of the motions impressed upon them, take directions, it may be parallel to each other, or in some other lines. The vein thus forms a species of truncated pyramid or cone, whose larger base is the orifice, and smaller the section of the fluid at its place of greatest contraction, a section which is often called the "section of contraction." This figure, and all the phenomena of contraction, are thus a consequence of the convergence of the several threads of water when they arrive at the orifice.

16. *Effects of the Contraction.*—When the orifice is in a thin plate, the contraction is completely external to the reservoir; it is thus clearly visible, can be, and, in fact, it has been, measured, as we shall mention directly. When the orifice is circular, the fluid vein, after having reached the minimum section, continues of the same transverse area, and is thus cylindrical in form, having a velocity very nearly equal to that due to the charge. The discharge will, therefore, be the product of this section by the velocity, so that the effect of the contraction is limited to the reduction of the value of the section which enters into the expression of the discharge. The flow will take place as if the actual orifice had been replaced by another whose diameter was equal to the "section of contraction," but in which supposed orifice no true contraction took place.

If to the orifice AB (Fig. 6) we attach a tube or cylindrical adjutage, the fluid threads will arrive at AB, converging, and therefore the fluid will contract at the entrance. Experiments prove that this contraction is identical with that of the thin plate; it will, however, be internal with respect to the mouth of the tube. Moreover, beyond the "section of contraction" the attraction of the sides of the tube occasions a dilatation of the fluid vein; the threads follow these sides, and issue parallel to each other and to the axis of the tube: so that the section of the vein at its exit is fully equal to that of the orifice in the side of the reservoir, but the velocity is not that due to the charge. If the flow was produced solely by the pressure of the fluid, probaby the velocity at the section of contraction would be that due to the charge, and would diminish in proportion as the transverse section of the vein enlarged, in consequence of that law or axiom of hydraulics:

namely—when an incompressible fluid is in motion, forming a continuous mass; then the velocity, at all its diverse sections, is inversely proportional to the area of the section. The diminution of velocity would cease when the fluid vein had, in diverging, reached the sides of the tube.

Now, since m is the ratio of the "section of contraction" to the section at the orifice, the velocity along the sides, and consequently at the mouth of the tube, would be $m\sqrt{2gH}$, and for the discharge we should have $S \times m\sqrt{2gH}$. In the orifices in a thin plate this discharge was also $mS\sqrt{2gH}$, giving the same discharge in both cases,—the only difference being, that in the latter area is effected, in the former the velocity: in the case of the added cylindrical adjutages it falls on the velocity. But the attractive action of the sides of the adjutage modifies this supposed state of things: not only does it cause the deviation of the fluid threads we have mentioned, but it is found that it also augments their velocity, so that the velocity of exit is greater than that given by the expression $m\sqrt{2gH}$; it will be $m'\sqrt{2gH}$, in which m' is greater than m, and the discharge will be $S \times m'\sqrt{2gH}$.

We thus see, that, in cylindrical adjutages, and, indeed, in adjutages in general, the effect of the contraction of the fluid vein is complicated with that of the attraction of the sides. Without being able to assign that which belongs to the first alone, it may be said that every internal contraction is connected with the diminution of velocity; and that every external contraction produces a diminution of section.

17. *Form and Dimensions of the Contracted Vein of the Fluid.*—Let us next examine the form that it gives to the fluid vein issuing from an orifice, in the simple case of a circular orifice in a thin plate, truly plane. Everything being symmetrical around the different points of the orifice, the direction, as well as the velocity of the molecules, the contracted vein ought also to be of a symmetrical form, and, consequently, a solid of revolution—a conoidal figure. It is actually so according to the observations that have been made, and which the figure AB *ab* (Fig. 7) represents. Beyond *ab* the contraction ceases, and the vein continues sensibly cylindrical for a certain length, until the resistance of the air and other causes entirely destroy this form.

The earlier measurements that have been made give to the three principal dimensions AB, *ab*, and CD, the ratio of the numbers 1.00, 0.79, and 0.39. The length of the contracted vein would thus be about half the diameter of the smaller section, and 0.39 of the larger, that is, of the orifice.

18. Michelotti, from a mean of more recent experiments on a large scale, has adopted 1.00, 0.787, 0.498 : these D'Aubuisson follows. The ratio of the diameters AB and *ab* being thus 1 to 0.787, that of the sections is 1 to ($0.787^2 =$) 0.619, that, namely, of the squares of the former numbers; thus, if s be the "contracted section," and S that of the orifice, we shall have—

$$s = 0.619\ S,$$

and, consequently, the discharge in one second, §§ 14 and 16—

$$s\sqrt{2g\text{H}},\ \text{or}\ 0.619\ S\sqrt{2g\text{H}},$$

so that the value of m, or the "coefficient of contraction," as determined by actual measurement, is, at the mean, equal to 0.619, being a little less than that which results from experiments on the discharge.

If the velocity at the passage of the "section of contraction" was actually that due to the charge, and that the flow took place through an adjutage of the exact form of the contracted vein, and that in the expression for the discharge the area s of the exterior orifice of this adjutage, taken at the extremity, were introduced, then the calculated would be equal to the actual discharge, and the coefficient of the reduction of the one to the other would be equal to unity; and Michelotti, in one of his experiments in which he employed a cycloidal adjutage, has reached 0.984. It is very probable he would have actually reached 1, if this form had more accurately been adapted to that of the fluid vein, and if the resistance of the air had not somewhat retarded the motion.

19. *Flow of Water through an Orifice in a Thin Plate.*—We come now to the direct determination of the coefficient for reducing the theoretical to the actual discharge. For this purpose it is necessary to gauge with care the volume of water discharged in a given time under a constant charge, from which we deduce the flow in one second, or the actual discharge; and, dividing this by the theoretic discharge for the same head and same orifice, the quotient is the *coefficient* required.

EXAMPLE.—Thus with a head of 4.012 feet we have a velocity of 16.07 feet per second; and the diameter of the orifice being 3.185 inches, we have its area equal to $3.185^2 \times 0.7854 = 7.97$ square inches, and $\frac{7.97}{144} = 0.0554$, its value in square feet; this, multiplied into the velocity of the water, gives the volume of the prism or cylinder equal to that of the water discharged; that is (§ 13), $0.0554 \times 16.07 = 0.8903$ cubic feet per second. But having found that in 1½ minutes the actual discharge is 49.68 cubic feet, and reducing this to its value for 1 sec. by dividing by 90, we obtain $\frac{49.68}{90} = 0.552$ cubic feet as the discharge in 1 sec.; hence, dividing the actual by the theoretic discharge, we find for the *coefficient* $\frac{0.552}{0.8903} = 0.620$.

Many hydraulicians have for a long time been engaged in its determination. The following Table, from D'Aubuisson, gives the principal results obtained by experiments up to the present time, and which, having been made under favourable circumstances, are generally received. They include circular, square, and rectangular orifices :—

TABLES *of the Results of Experiments to determine the* "*Coefficient of Contraction.*"

CIRCULAR ORIFICES.			
Observers.	Diameters.	Charges.	Coefficient.
	Feet.	Feet.	
Mariotte, .	0.0223	5.8712	0.692
Do.	0.0223	25.9120	0.692
Castel, . .	0.0328	2.1320	0.673
Do.	0.0328	1.0168	0.654
Do.	0.0492	0.4526	0.632
Do.	0.0492	0.9840	0.617
Eytelwein,	0.0856	2.3714	0.618
Bossut, . .	0.0889	4.2640	0.619
Michelotti, .	0.0889	7.3144	0.618
Castel, . .	0.0984	0.5510	0.629
Venturi, . .	0.1345	2.8864	0.622
Bossut, . .	0.1771	12.4968	0.618
Michelotti, .	0.1771	7.2160	0.607
Do.	0.2657	7.3472	0.613
Do.	0.2657	12.4968	0.612
Do.	0.2657	22.1728	0.597 ?
Do.	0.5314	6.9208	0.619
Do.	0.5314	12.0048	0.619

SQUARE ORIFICES.

Observers.	Side of Square.	Charges.	Coefficient.
	Feet.	Feet.	
Castel, . .	0.0032	0.1640	0.655
Bossut, . .	0.0885	12.5000	0.616
Michelotti, .	0.0885	12.5000	0.607
Do.	0.0885	22.4078	0.606
Bossut, . .	0.1771	12.5000	0.618
Michelotti, .	0.1771	7.3472	0.603
Do.	0.1771	12.5624	0.603
Do.	0.1771	22.2384	0.602
Do.	0.2689	7.3489	0.616
Do.	0.2656	12.5624	0.619
Do.	0.2656	22.3700	0.616

RECTANGULAR ORIFICES.

Rectangle.		Charges.	Coefficients.
Height.	Base.		
Feet.	Feet.	Feet.	
0.0301	0.0606	1.0824	0.620
0.0301	0.1213	1.0824	0.620
0.0301	0.2423	1.0824	0.621
0.0301	0.4847	1.0824	0.626

20. The experiments of Michelotti were carried on about three miles from Turin, at an hydraulic establishment constructed for experimental purposes, consisting of a building 26 feet high, supplied with water from the River Dora by a canal of derivation. The internal dimension was a square of 3 feet 2¼ inches; on one of the sides was arranged a series of adjutages at the different depths deemed expedient, and upon the surface of the ground were arranged the different receptacles for the gauging of the actual discharges. It may be remarked upon his part of the Table, than the coefficients obtained from the large orifices are higher that the others, and this contrary to the rule that would be deduced from the experiments in general.

21. In order to place the subject of the variation in the value of the coefficient, under different circumstances of area and charge, in a clear point of view, the following Table of MM. Poncelet and Lesbros' experiments at Metz, in the north

of France, in 1826 and 1827, is given. In these experiments the orifices are rectangular, and all of the same breadth—namely, $0^{m}.20 = 0.656$ ft.; the heights were successively 0.656, 0.328, 0.164, 0.098, 0.065, and 0.0328 feet. The charges extended from 0.33 feet to 5.58 feet. With each orifice they repeated the experiments, taking them with 8 or 10 charges, from the smallest, to the highest that the apparatus admitted, calculating the corresponding coefficients. They then took the charges for the abscissæ, and these coefficients for the ordinates of a curve constructed for each orifice, and by its aid they determined the ordinates,—that is, the coefficients intermediate to those directly determined by experiment; and thus gave a very extended Table, from which the following is taken:—

TABLE *showing the Results of Experiments to determine the Variation in the Value of the Coefficient of Contraction.*

Charge on Centre of Orifice.	HEIGHT OF THE ORIFICES.						Difference of maximum and minimum coefficients.
	Feet. 0.656	Feet. 0.328	Feet. 0.164	Feet. 0.098	Feet. 0.065	Feet. 0.032	
Feet.							
0.032						0.709*	
0.065					0.660	0.698	
0.098				0.638	0.660*	0.691	
0.131			0.612	0.640	0.659	0.685	
0.164			0.617	0.640	0.659	0.682	
0.196		0.590	0.622	0.640*	0.658	0.678	
0.262		0.600	0.626	0.639	0.657	0.671	
0.328		0.605	0.628	0.638	0.655	0.667	
0.393	0.572	0.609	0.630	0.637	0.654	0.664	0.092
0.492	0.585	0.611	0.631	0.635	0.653	0.660	0.075
0.656	0.592	0.613	0.634*	0.634	0.650	0.655	0.063
0.984	0.598	0.616	0.632	0.632	0.645	0.650	0.052
1.312	0.600	0.617*	0.631	0.631	0.642	0.647	0.047
1.640	0.602	0.617	0.631	0.630	0.640	0.643	0.041
2.296	0.604	0.616	0.629	0.629	0.637	0.638	0.034
3.281	0.605*	0.615	0.627	0.627	0.632	0.627	0.027
4.264	0.604	0.613	0.623	0.623	0.625	0.621	0.021
5.248	0.602	0.611	0.619	0.619	0.618	0.616	0.017
6.562	0.601	0.607	0.613	0.613	0.613	0.613	0.012
9.843	0.601	0.603	0.606	0.607	0.608	0.609	0.008

Fig. 8 illustrates this method of interpolation. From the point O, the several charges are laid off on the line ON, as OX, OX_1, &c., and the corresponding coefficients XY, X_1Y_1, &c.; and the curve being traced through Y, Y_1, Y_2, &c., we can obtain the coefficient proper to any charge as Ox, by drawing the perpendicular xy terminating in the curve.

22. All the numbers contained in this Table are the several values of the coefficient m in the formula $Q = mS\sqrt{2g\text{H}}$. But those in each column above the transverse line are not the true coefficients for the reduction of the theoretic to the actual value, as will be shown hereafter.

Casting the eye over each column, we may see that the coefficients increase as the charges are greater, but up to a certain point only, although the charge still increases: an asterisk in each column points out the respective maxima. It may also be observed, that the coefficients become more nearly equal in each column as the charges increase,—the bottom line of figures, in which the charge was $3^{\text{m}} = 9.84$ feet, being almost identical in each column.

23. This Table, although constructed from experiments on rectangular orifices, can yet be extended to those of all other forms,—the height of the rectangle, as given in the Table, corresponding to the smaller dimension of the orifice made use of; for it is admitted that the discharge is altogether independent of the figure of the orifice when the area is constant, provided only that this figure has no re-entrant angles.

24. Although these experiments are on a considerable scale, yet there are some cases in practice in which the discharge is twenty or thirty times greater. Such are the sluices in lock-gates on canals of navigation. It is a matter of importance to determine directly the coefficient of discharge for them.

TABLE *showing the Value of the "Coefficient of Contraction" in large Sluices: Canal Languedoc.*

SLUICE.—Width 4.25 ft.		Charge upon the Centre.	Discharge in 1 sec.	Coefficient.
Area.	Height.			
Square Feet.	Feet.	Feet.	Cubic Feet.	
7.7442	1.804	14.550	145.3056	0.613
6.9928	1.640	6.628	92.6438	0.641
6.9928	1.640	6.245	88.2288	0.629
6.4664	1.508	12.874	138.6302	0.641
6.7237	1.574	13.582	128.7759	0.647
6.7237	1.574	6.392	83.9551	0.616
6.7237	1.574	6.215	79.8580	0.594
6.7172	1.574	6.478	85.2266	0.621

The mean coefficient, 0.625, is rather greater than that found by Poncelet (§ 21), which is readily explained, as the flow of water did not take place as if in a thin plate, the contraction being suppressed on some parts of the boundary. The woodwork which surrounded the sluiceway was = 0.8856 feet thick, and on the sill was even = 1.771 ft. Thus, when the sluice was raised but a small height, the contraction nearly ceased on four sides, and the coefficient was considerably increased. For example, when the paddle was raised only 0.393 ft., it gave a coefficient of 0.803; when raised 1.51 ft., it was 0.641.

25. *Particular Cases in which the Contraction is suppressed on one or more sides of the Orifice.*—In all the different cases treated of hitherto, it has been assumed that the fluid arrived at the orifice from all parts equally, but frequently this is not so. For example (Fig. 9), when a rectangular orifice is at the bottom of a vertical plate, and its inferior edge is on the level of the bottom of the vessel or reservoir, the contraction is then destroyed on that side, the particles of water being compelled to take a direction parallel to the side of the vessel; and, consequently, the discharge is increased. The question arises, therefore, how much will the discharge be augmented by the suppression of the contraction for a certain length of the periphery of the orifice?

The following Table gives the result of experiments instituted with the view of determining this point. The orifice was rectangular, 0.177 feet in base, and 0.089 feet in height. The plates, which were attached, sometimes on one side, sometimes on two or three of the sides, were 0.22 feet long; that is, they advanced this much into the reservoir. The flow was produced by charges from 6.56 feet to 22.56 feet in height:—

TABLE *showing the Increase of the "Coefficient of Contraction" by its Suppression on Part of the Sides.*

Portion of Orifice without Contraction.	Coefficient.	Ratio of Increase.
0	0.608	1.000
$\frac{1}{6}$	0.620	1.020
$\frac{2}{6}$	0.637	1.049
$\frac{3}{6}$	0.659	1.085
$\frac{4}{6}$	0.680	1.119
$\frac{5}{6}$	0.692	1.139

26. In this Table the last column has for its unit the discharge when the orifice is perfectly free: the numbers, therefore, indicate the increase in the coefficients, and, consequently, in the discharges. The formula deduced by M. Bidone, the experimenter, is $1 + 0.152 \frac{n}{p}$, in which n represents the length of the part of the perimeter in which the contraction is suppressed, and p the perimeter of the orifice. The greatest error of this formula being but the $\frac{1}{39}$th part, it may be used for the value of the discharge when, in the case of rectangular orifices, there is no contraction on part of the boundary, and the actual discharge then is $mS\sqrt{gH}\,(1 + 0.152 \frac{n}{p})$.

27. *Orifices in plates not being true planes.*—The supposition hitherto has always been that the sides or plates in which the orifices were placed were truly plane; they may, notwithstanding, be of very different surfaces. In order to have a clear idea of the effect which this alteration produces upon the flow, it is necessary to recall to mind that if the threads of the fluid vein did arrive at the orifice mutually parallel, the actual discharge would be equal to the theoretic, and that it is less only by reason of the oblique directions in which they meet, from which necessarily results a destruction of part of the acquired motion at the point of contact. If, therefore, we imagine around the orifice a spherical surface of a radius equal to that of the sphere of action of the orifice, and this surface terminated by the sides of the vessel, then it must be intersected on every point, and in directions nearly perpendicular, by the threads of the issuing fluid (Figs. 10 and 11); and the larger the portion of the sphere this surface may be, and the more oblique, or even opposite, to one another, the threads of the fluid arrive at it, then the more the motion is destroyed at the entrance of the orifice, and the less the discharge is found to be. When the sides are developed in one plane, then this supposed surface is a hemisphere (Fig. 6), and the coefficient of the particular case is given above, p. 18, § 21. But if they are disposed in the form of a funnel, or, if simply concave, towards the interior of the vessel, then the surface of this sphere is of less extent, and the discharge more considerable—not, however, following exactly the inverse proportion of the spherical surface. If, on the other hand, the side is convex, the discharge is diminished, and it will be less still in the case represented in Fig. 10. Lastly, it will be at its minimum if the supposed

surface should become an entire sphere; and this would happen if it was possible to carry an orifice into the midst of the mass of the fluid enclosed in the vessel.

28. Borda has succeeded in realizing this case almost completely. He has introduced into a vessel (Fig. 11) a tube of tin 0.443 feet long and 0.105 feet in diameter, and under a charge of 0.82 feet he has caused the flow to take place, so that the effluent water did not touch the sides of the tube at all. The actual discharge has been only 0.515 of the theoretic, and from various circumstances Borda was led to think that he might have reduced it to 0.50.

Having subsequently surrounded the orifice of the entry of the tube with a border or rim, and having thus reduced it to the condition of being in a perfectly plane plate, although in the centre of the fluid mass, he found the coefficient rise to 0.626. The same result might be obtained by employing a simple tube, but formed of a thick material. Fig. 12 shows the manner in which the fluid bends around the exterior edge, and enters the tube without touching the internal sides,—the thickness being about $\frac{1}{12}$ of an inch, or 0.0065 feet, and the edges cut truly square: thus all that part of the sides within the exterior periphery is, as far as the discharge is concerned, as if totally removed; and it is this external diameter that should be introduced in all calculations relative to internal adjutages. By taking it, M. Bidone has found, from two experiments in which the effluent fluid did not touch the sides, that the coefficient was nearly 0.50,—that is, the section of contraction was half the orifice taken at the external circumference.

29. Thus 0.50 and 1.00 will express the limits of the coefficients of contraction,—limits to which they may approach very nearly, but which they can never actually attain. For orifices in a plate truly plane it does not descend below 0.60, or rise much above 0.70; and in ordinary practice it ranges between 0.60 and 0.64. As a mean term, 0.62 is generally taken; so that—

$$Q = mS\sqrt{2g\mathrm{H}} = 0.62S\sqrt{2g\mathrm{H}} = 4.96S\sqrt{\mathrm{H}};$$

and if the orifice be circular of a diameter d, the area is expressed by $d^2 \times 0.7854 = S$, and $Q = 3.9\, d^2\sqrt{\mathrm{H}}$. For greater exactness in the coefficient, recourse should, however, be had to the Table, page 18, § 21.

30. *Effects on the Discharge when the Fluid has Velocity antecedently.*—If the water contained in the reservoir, instead of being in a state of repose, was moving in the direction of the orifice,—as when the vessel, having but a relatively small section, has a supply of water brought into it, and flowing directly up to the plate or side in which the orifice is opened,—then the particles of the fluid would issue, not only in virtue of the pressure exerted by the fluid mass which is above it, but with the additional velocity that they had when they entered into the sphere of action of the orifice; we must, therefore, add to the actual charge measuring the pressure a new term, which will be the height due to this supposed velocity of arrival. Thus, if u represent this velocity, we shall have (since $\frac{u^2}{2g}$ is the height producing the velocity u) the expression—

$$Q = mS\sqrt{2g\left(\mathrm{H}+\frac{u^2}{2g}\right)} = mS\sqrt{2g\mathrm{H}+u^2}.$$

31. For example:—A basin 20 mètres long, and 2^{m} wide, and 1^{m} deep, has at one extremity a plank in which is cut a rectangular orifice $0^{m}.55$ wide, and $0^{m}.36$ deep; its sill or lower edge is $0^{m}.91$ below the constant level at which the water in the basin remains. At the other end it receives a supply of water: what then is the discharge?

$$S = 0.55 \times 0.36 = 0^{mm}.198$$
$$\mathrm{H} = 0.91 - \tfrac{1}{2}\,0.36 = 0^{m}.73$$
$$m = 0.600$$

the value of m being deduced by extending the Table, § 21.

The value of Q, taken first in neglecting u, will be—

$$0.600 \times 0.198\sqrt{2g \times 0.73} = 0^{mmm}.4496 = 15.878 \text{ cb. ft.}$$

Now, when water flows in a canal with a mean velocity u, we evidently have, s being the transverse section, $Q = su$.

Then, dividing by s, we have $\frac{Q}{s} = u$, or as $s = 2^{m}$

$$\frac{0.4496}{2} = 0^{m}.2248 = u,$$

and, therefore, $u^2 = 0.0505$.

And putting this value in the general expression—

$$Q = 0.6 \times 0.198 \sqrt{14.321 + 0.0505} = 0^{\mathrm{mmm}}.45025 = 15.9 \text{ cb. ft.}$$

The French measures have been here retained, to afford an example for the reduction of the mètre into English feet, 1 mètre being 3.281 ft.

32. *Flow of water with cylindrical adjutages.*—We have already seen (§ 16) that the addition of a tube to an orifice in a thin plate gives a discharge larger than that through an orifice in a thin plate; but in order that it should produce this effect it is necessary that the water entirely fill the area of the external mouth of the tube, and this is generally the case when the length of the tube is three or four times greater than its diameter: if it be less than this, it frequently occurs that the fluid vein which is contracted at the entrance does not enlarge so as to fill the interior of the tube; the flow in that case takes place as if in a thin plate, and this is always the case when the length of the tube is less than the length of the contracted vein, and, therefore, but half the diameter, or less.

33. TABLE *showing the increase in the "Coefficient of Contraction" by the cylindrical Adjutage.*

Observers.	Adjutage.		Charge.	Coefficient.
	Diameter.	Length.		
	Feet.	Feet.	Feet.	
Castel, .	0.0508	0.1312	0.656	0.827
Do.	0.0508	0.1312	1.574	0.829
Do.	0.0508	0.1312	3.247	0.829
Do.	0.0508	0.1312	6.560	0.829
Do.	0.0508	0.1312	9.938	0.830
Bossut, .	0.0754	0.1771	2.132	0.788
Do.	0.0754	0.1771	4.067	0.787
Eytelwein,	0.0852	0.2558	2.361	0.821
Bossut, .	0.0885	0.1344	12.628	0.804
Do.	0.0885	0.1771	12.693	0.804
Do.	0.0885	0.3542	12.857	0.804
Venturi, .	0.1344	0.4198	2.886	0.822
Michelotti,	0.2656	0.7084	7.150	0.815
	Square.			
Do.	0.2656	0.7084	12.464	0.803
Do.	0.2656	0.7084	22.008	0.803

34. The mean of these coefficients gives 0.817, its value is generally taken as 0.82, so that we have the following formulæ:—

$$Q = 0.82 S\sqrt{2g\mathrm{H}} = 6.56 S\sqrt{\mathrm{H}};$$

and if d be the diameter of a circular orifice—

$$Q = 0.82 \frac{\pi}{4} d^2\sqrt{2g}\sqrt{H} = 5.152 d^2\sqrt{\mathrm{H}}.$$

35. In the case when the jet issues with the tube full, in threads parallel to the axis of the orifice, and when, consequently, the section is equal to that of the orifice, the diminution of the discharge can only occur from a diminution of the velocity; and the ratio of the actual to the theoretic discharge is the same as that of the actual to the theoretic velocity.

TABLE *showing the identity of the "Coefficients of Discharge" and Velocity, with the cylindrical Adjutage.*

Observers.	Coefficient of the	
	Velocity.	Discharge.
Venturi,	0.824	0.822
Castel, .	0.832	0.827
Castel, .	0.832	0.829

The three quantities measured were the "charge" on the centre of the tube, the velocity computed by measuring ordinate and abscissa, as in § 8, and the volume discharged. The velocity due to the charge, compared with that so computed, gives the second column, and the product of the area of the tube into the velocity due to the charge, compared with the discharge, gives the third; that is—

$V\ (=\sqrt{2g\mathrm{H}})$: computed Velocity, :: 1 : 0.824, and
$S \times V$: Discharge . . . :: 1 : 0.822.

We must, therefore, conclude that the velocity of a jet of water at the extremity of a cylindrical adjutage is equal to 0.82 of that due to the charge, and that the "head" due to that velocity is but 0.67 of the actual "head" of the reservoir; that is $(0.82)^2$, because the "heads" or charges are as the squares of the velocities.

36. As to the cause of this increase of the coefficient from 0.62 to 0.82, D'Aubuisson ascribes it to the attraction of the

sides of the tube and the divergence of the fluid threads. After they have come in contact with them, they are forcibly retained by the molecular attraction, such as that which causes the rise of fluids in capillary tubes: by this same force the outer threads draw after them the inner, and so all the vein issues with a full tube, and passes with an increased velocity through the contracted section. The immediate cause is the contact; and every circumstance which favours that tends to produce an augmentation of the coefficient.

37. *Flow of water through conical converging adjutages.*—Conical adjutages, properly so called, that is, those which are slightly converging to a point exterior to the reservoir, augment the discharge still more than the preceding. They give jets of great regularity, and throw the water to a greater distance or height, and are hence frequently used in practice: the effects vary with the angle of convergence of the sides.

Two distinct contractions of the fluid vein take place with this adjutage—one interiorly, or at the entrance of the adjutage, which diminishes the velocity due to the charge; the other at the exterior; in consequence of which the true section of the fluid vein is slightly less than the area of the external mouth of the adjutage.

If, therefore, we put S for the section of the external orifice, V for the velocity due to the charge, the actual discharge will be expressed by $nS \times n'V = nn'SV$, the two coefficients n and n' must be found by experiment, n being the ratio of the section of the fluid at its least diameter to that of the orifice, or the coefficient of the exterior contraction, and n' that of the actual velocity to the theoretic, or the *coefficient* of *the velocity*, and nn' their product, is the ratio of the actual discharge to the theoretic, or the *coefficient* of *the discharge*. The knowledge of these two last is of practical importance in the case of jets of water, as in fountains and fire-engines.

38. In order to determine the coefficients above mentioned, and especially to ascertain the angle of convergence that gives the maximum discharge, experiments were undertaken with a number of adjutages successively (Fig. 12^a), in all of which the diameter of the orifice of final issue and the length of the adjutage remained constant; but in each the diameter of entrance, and consequently the angle of convergence, was increased. The flow of the water was produced with different charges with each of these varied adjutages.

At every experiment the discharge was determined by actual gauging, and the velocity of issue by the method of the parabola, given above (§ 8). The discharge, divided by SV, gave the product nn', and the observed velocity, divided by V (= $\sqrt{2gH}$), gave n'.

The series of the numbers nn' showed the discharge corresponding to each angle of convergence, and consequently the angle of maximum discharge, and the series of n', marked the progression by which the velocities increased.

39. The same adjutage, under charges which varied from 0.69 feet to 9.94 feet, or from 1 to 14, always gave discharges proportional to $\sqrt{H}$, and therefore the coefficient, or nn', has been, *q.p.*, the same also. A very small increase may be observed with the higher charges. With respect to the coefficients of the velocity, they also should have been found constant but for the resistance of the air. Now, this resistance diminishing the throw of the jet, and that in proportion as the charge is greater, we should expect in the coefficients calculated from it, a decrease augmenting with the charge,—although, at the same time, there was no actual diminution in the velocity with which the fluid issued, or tended to issue.

TABLE *showing the "Coefficients with Conical Converging Adjutages," the angle of Convergence being that giving the maximum Discharge, as determined in the next Table.*

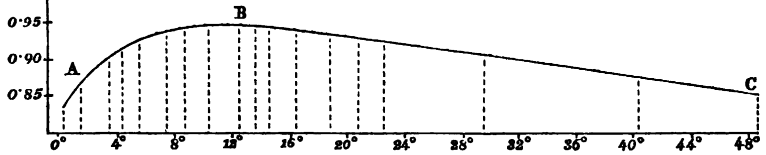
Charge.	Coefficient of the	
	Discharge = nn'.	Velocity = n'.
Adjutage, . . . 0.0508 feet diameter.		
Feet.		
0.705	0.946	0.963
1.584	0.946	0.966
3.253	0.946	0.963
4.893	0.947	0.966
6.579	0.946	0.956
9.938	0.947	
Adjutage, . . . 0.0656 feet diameter.		
0.692	0.956	0.966
1.584	0.957	0.968
3.263	0.955	0.965
4.913	0.956	0.962
6.586	0.956	0.959
9.938	0.957	

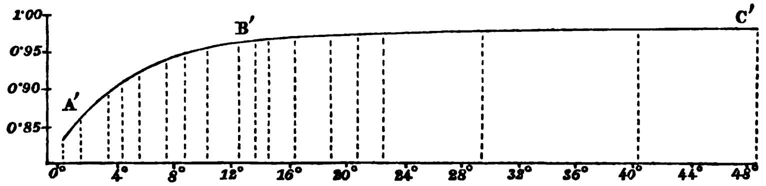
Let us, in the next place, compare together the coefficients both of the discharges and of the velocities obtained, with the different adjutages of one and the same series,—adjutages which only differed in the angle of their convergence. Each coefficient is derived from a mean of five or six experiments taken with different charges, very nearly the same as those put down in the preceding Table.

TABLE *showing the variation of the Coefficients of Discharge and Velocity with Conical Converging Adjutages at different angles.*

Angle of Convergence.	Coefficient of the		Angle of Convergence.	Coefficient of the	
	Discharge.	Velocity.		Discharge.	Velocity.
Diameter, . . 0.0508 feet.			Diameter, . . 0.0656 feet.		
0° 0′	0.829	0.830			
1° 36′	0.866	0.866			
3° 10′	0.895	0.894	2° 50′	0.914	0.906
4° 10′	0.912	0.910			
5° 26′	0.924	0.920	5° 26′	0.930	0.928
7° 52′	0.929	0.931	6° 54′	0.938	0.938
8° 58′	0.934	0.942			
10° 20′	0.938	0.950	10° 30′	0.945	0.953
12° 4′	0.942	0.955	12° 10′	0.949	0.957
13° 24′	0.946	0.962	13° 40′	0.956	0.964
14° 28′	0.941	0.966	15° 2′	0.949	0.967
16° 36′	0.938	0.971			
19° 28′	0.924	0.970	18° 10′	0.939	0.970
21° 0′	0.918	0.971			
23° 0′	0.913	0.974	23° 4′	0.930	0.973
29° 58′	0.896	0.975	33° 52′	0.920	0.979
40° 20′	0.869	0.980			
48° 50′	0.847	0.984			

The facts established in the first Table, Diameter, 0.05 feet, are represented graphically in the two following engravings; the upper referring to the discharges, the lower to the velocities. In each the horizontal line indicates degrees, extending from 0° to 48°, and having 0.3 inch equal to 4°. On this line the degrees of convergence of the adjutages are laid off as abscissæ from 0°, the coefficients corresponding being laid off on the dotted vertical lines as ordinates; the scales for these, in which 1.00 = 4 inches nearly, are given at the left-hand side commencing at 0.80, which is set to the horizontal line, as none of the coefficients were less. The curve line A, B, C, is drawn through the termination of the ordinates of discharge, and A′, B′, C′, through those of the velocity.





40. It follows, both from the Table and the wood engraving, 1st, That, for the same orifice, and with the same constant charge, the actual discharge, commencing with 0.83 of the theoretical, increases gradually in proportion as the angle of convergence increases up to 13½°, near B, at which the cofficient of discharge is 0.95: beyond this angle it diminishes at first slowly, as do all variables, about the maximum. At 20° the coefficient is yet as high as 0.92 : subsequently the diminution becomes more and more rapid, and terminates as low as 0.65, which is the coefficient of discharge through a thin plate,—this last being the ultimate position of converging adjutages, that, namely, in which the angle of convergence has attained its maximum, or 180°. Thus, then, we have for the maximum discharge an angle of convergence of between 13° and 14°.

2ndly. In looking at the coefficients and ordinates of velocity, we see them also increasing from 0°, nearly as those of the discharge, up to 10°; after that they increase more rapidly; and when beyond the angle of maximum discharge, while the coefficients of discharge diminish, they continue to augment and approach their limit of unity. They are very nearly equal to unity at 50°, and even at 40° are not far from it. Thus, conical adjutages may, by varying the angle of convergence, be made to form a series or progression, whose first term is the cylindrical adjutage, and last the orifice in a thin plate; the velocity of projection, increasing with the angle of convergence, will vary from that of the tube additional, up to that of the simple orifice in a thin plate; that is to say, from $0.82 \times \sqrt{2gH}$ up to $1 \times \sqrt{2gH}$.

3rdly. If we compare the coefficients of discharge with those of the velocity,—that is, the successive values of $n \times n'$ and n', and divide the former by the latter,—we shall have the values of n, or the coefficients of the exterior contraction. From the angle 0° up to 10°, n is sensibly equal to 1, and, consequently, no such contraction was present in the experiment; and, notwithstanding the convergence of the sides, the fluid particles issued, *q.p.*, parallel to the axis of the cone. Beyond 10°, however, the contraction becomes apparent; it reduces more and more the section of the vein, and terminates by bringing it to an equality with that of the orifice in the thin plate, as is shown here:—

TABLE *showing the value of n, the Coefficient of Exterior Contraction, with different angles of Convergence.*

Angle.	n, or $\frac{nn'}{n''}$.
8°	1.00
15°	0.98
20°	0.95
30°	0.92
40°	0.89
50°	0.85
180°	0.65

In these experiments the length of the conical adjutage was fixed at about 2½ times the exterior diameter (Fig. 12^b); thus it was 0.1312 feet for those of 0.0508 feet, and 0.1640 feet for those of 0.0656 feet, in order to avoid, as far as possible, complicating the results with the effect of the friction against the sides, in this following the analogy of the cylindrical adjutage, in which experience proves that, with respect to discharge, they produce their full effects most certainly when the length equals 2½ times the diameter.

41. As to those very large conical adjutages, or rather truncated pyramidal tubes, which in manufactories discharge water upon mill-wheels, three very valuable experiments, made at a mill on the canal of Languedoc, are given by the engineer,

Lespinasse. They were, in this case, formed by the sides of a rectangular pyramid, whose length was 9.59 feet; rectangle of large end, 2.4 feet by 3.2 feet; at the smaller, 0.443 feet by 0.623 feet.

The opposite faces made angles of 11° 38′ and 15° 18′; the charge was 9.6 feet:—

TABLE *showing the "Coefficient" of Discharge with very large Converging Mill-sluices.*

Discharge.	Coefficient.
Cubic Feet.	
6.767	0.987
6.692	0.976
6.714	0.979

We see, then, how very little such adjutages diminish the discharge: that which they give is only one or two hundredths below the theoretic discharge.

42. *Conical adjutages diverging.*—This adjutage, of all others, gives the largest discharge. It may be described as a truncated cone attached to a reservoir by its smaller diameter, and of which the exterior mouth is consequently greater than that of the entry of the water. Although not much in practical use, they present phenomena of such interest as to deserve some notice.

The property they have of increasing the discharge was known to the ancient Romans: some of the citizens, to whom had been granted the privilege of having a certain quantity of water from the public reservoirs, found, by using these adjutages, the means of increasing their supply; and the fraud became so extensive that their use was forbidden by law, except when the distance from the reservoir was not less than about 52 feet. Venturi is the experimenter to whom we are chiefly indebted for information respecting this particular adjutage.

43. Those which he made use of carried a mouth-piece, ABCD (Fig. 15), not unlike the form of the contracted vein, AB being equal to 0.1332 feet, and CD equal to 0.1109 feet. The body of the adjutage varied in length and in its divergence: this last was measured by the angle contained between the sides EG and FD, supposed prolonged until they meet. These adjutages were attached to a reservoir maintained at a

constant level; the flow took place under a constant charge of 2.89 feet; and the time required to fill a vessel of 4.838 cubic feet was observed.

The following Table gives the result of his principal observations, premising that the time corresponding to the theoretic velocity was 25.49 seconds:—

TABLE *showing the variation of the "Coefficient" of the Discharge with Conical Diverging Adjutages at different angles and lengths*

Adjutage.		Time of Flowing.	Coefficient.
Angle.	Length.		
	Feet.	Seconds.	
3° 30′	0.364	27.5	0.93
4° 38′	1.095	21	1.21
4° 38′	1.508	21	1.21
4° 38′	1.508	19	1.34
5° 44′	0.577	25	1.02
5° 44′	0.193	31	0.82
10° 16′	0.865	28	0.91
10° 16′	0.147	28	0.91
14° 14′	0.147	43	0.61

Venturi has drawn the conclusion that the adjutages of maximum discharge should have a length of nine times the diameter of the smaller base, and an angle of divergence equal to 5° 6′: it is represented in Fig. 15. This, he adds, would give a discharge 2.4 times greater than the orifice in a thin plate, and 1.46 times greater than the theoretic discharge.

44. *Flow of water under very small charges.*—When the charge over the centre of the orifice is very small compared with the vertical depth of the orifice itself, then the mean velocity of the different threads of the fluid vein—that is to say, the velocity which, being multiplied by the area of the orifice, gives the actual discharge—is no longer that of the central thread. It differs from it in proportion as the charge is less: its true value will be about the hundredth part less when the charge is equal to the depth of the orifice, and by about the thousandth part when equal to three times that depth. Let us examine what theory teaches under this head, and first, of the law which it assigns for the velocity of the fluid threads in proportion as their depth below the surface of the water in the reservoir at which they issue increases.

45. *Velocity of any fluid thread whatever.*—Let A (Fig. 16) be the level of the surface of water in a vessel, and upon the face AB—which, for greater simplicity, we suppose vertical—let us imagine a series of very small orifices placed one below the other, and of which that at B is the lowest, and putting H for the height AB, the velocity of the jet issuing from B will be expressed by $\sqrt{2g\mathrm{H}}$; if we make the line BC equal to this quantity, it will represent this velocity; for any other point P, taken below the surface of the water at a depth equal to AP or x, the velocity of the issue will be represented by the line PM = $\sqrt{2gx}$, and calling this y, we shall have—

$$y = \sqrt{2gx}.$$

If through all the points M so found we draw a curve-line, they will be the ordinates, and the heights AP or x will be the abscissæ; and because $y^2 = 2gx$, this curve will be a parabola, having $2g$ or 64.4 feet for its parameter, and thus we have this proposition:—The velocity of a fluid thread issuing from a reservoir, at any point, is equal to the ordinate of a parabola whose parameter is equal to twice the measure of the force of gravity, and the depth below the surface of the water is the abscissa.

46. *Volume discharged.*—Let us, in the next place, suppose that, instead of this series of small orifices in the face AB, we had opened a rectangular slit, whose width was l, and inquire the discharge in this case, and, for simplicity, let us at first omit the "contraction" and its effects. Suppose this opening divided into a number of elementary rectangles by horizontal lines very close to each other, the volume of water which will issue from each of them in one second, or its discharge, will evidently be equal to the volume of a prism, whose base is the elementary rectangle, and height the velocity of issue; i. e. the ordinate corresponding. The sum of all the small prisms, or the total discharge, will also be equal evidently to another prism, whose base is the parabolic segment ABCMA, and its height or thickness equal to l, the width of the opening.

Now, from the well-known property of the parabola, this segment is two-thirds of the rectangle ABCK, whose area is AB × BC, which has been shown to be equal to* $H \times \sqrt{2gH}$;

---

* The italic capital H is used for the depth from the surface to the sill or bottom line; the italic h for the depth to the top of any rectangular orifice; and the capital H for the mean depth, or $\frac{H+h}{2}$.

and thus the discharge for the rectangular opening, whose height is H and width l:

$$\frac{2}{3} \times l \times H \times \sqrt{2gH} = \frac{2}{3} l \times \sqrt{2g} \times H . \sqrt{H}.$$

47. In the next place, let us seek to determine the discharge through a rectangular orifice opened in the same face, but only from B to D, and having, as before, the constant width l. Call the charge AD on the upper edge of the orifice h, the discharge of the opening from A to D will be—

$$\frac{2}{3} l \times h \times \sqrt{2gh} = \frac{2}{3} l \sqrt{2g} \times h \sqrt{h}.$$

Now it is evident that the discharge from the rectangular orifice, whose height is BD, will be the difference of the discharges by the two openings AB and AD, into l, and will, therefore, be expressed by—

$$\frac{2}{3} l \sqrt{2g} \left(H \sqrt{H} - h \sqrt{h}\right).$$

That which has been established in § 13 is, as $S = l(H-h)$,

$$l \sqrt{2g} \, . \sqrt{\mathrm{H}} \, . (H - h),$$

on the supposition, very nearly correct, that the velocity at the mean depth was the mean velocity.

48. *Mean velocity.*—Let us now return to (§ 44) the determination of the mean velocity, and first, that of the entire rectangular opening up to the surface. Let G be the point from which would issue the thread with this mean velocity. If we make AG = z, it will be expressed by $\sqrt{2gz}$, and this being multiplied into the area of the opening, $l \times H$, must give the total discharge, which we have already found to be expressed by—

$$\frac{2}{3} . l . H \sqrt{H} \sqrt{2g},$$

we shall therefore have—

$$l \times H \sqrt{2g} \times \sqrt{z} = \frac{2}{3} l \times H \times \sqrt{2g} \times \sqrt{H},$$

dividing each side by the common factors $l \times H \times \sqrt{2g}$, we have, squaring both sides,

$$z = \frac{4}{9} H;$$

and, consequently,

$$V = \sqrt{2g \frac{4}{9} H} = \frac{2}{3} \sqrt{2gH}.$$

Thus it appears that the mean velocity is equal to two-thirds of the velocity of the lowermost fillet; and so GH, which represents the former, is ⅔rds of BC, which, in like manner, represents the latter.

For the case of the rectangular orifice, whose depth is BD or $H - h$, we shall, in like manner, have the area expressed by $(H - h) \times l$, and the discharge, making z' the height due to the mean velocity, by—

$$(H - h) \times l \times \sqrt{2gz'} = \frac{2}{3} l \times \sqrt{2g}\,(H\sqrt{H} - h\sqrt{h}).$$

Dividing both sides by $l \times \sqrt{2g}$, we have—

$$(H - h) \times \sqrt{z'} = \frac{2}{3}(H\sqrt{H} - h\sqrt{h}),$$

and dividing by $H - h$, and squaring—

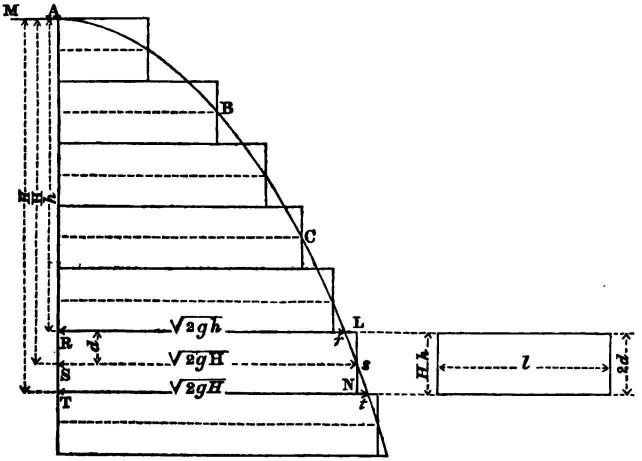
$$z' = \frac{4}{9}\left(\frac{H\sqrt{H} - h\sqrt{h}}{H - h}\right)^2.$$

and hence, the mean velocity,

$$V = \sqrt{2g} + \frac{2}{3} \times \frac{H\sqrt{H} - h\sqrt{h}}{H - h}.$$

In order to have a clear idea of the actual difference in the volume of water, as computed by the two formulæ, that, namely, which multiplies the velocity at the mean depth into the area of the orifice, which is the one almost always employed, and the true formula given in § 47, the following Table was calculated, in which the area of the sluice, as opened, is supposed to be 2 ft. deep by 4 ft. wide, and placed in ten different positions as to depth from the surface. In the first, the upper line of the orifice is on the level of the water surface, and therefore $h = 0$; in the second, 2 ft. below it, and so on successively 2 feet deeper, until $h = 18$, and $H = 20$; the several values of H will therefore be 1, 3, 5, &c., up to 19.

The wood engraving illustrates this arrangement up to the seventh position, and also shows graphically the reason



of the difference of calculating by the two formulæ. The level of the water surface is supposed to be on the line MA, and the vertical line ARST, the edge of the wall or plate seen in the transverse section, in which the orifice or sluice-way is supposed to be opened; RT = 2 ft. is the sixth position in which the opening is placed, and, therefore, $h = 10$, $H = 12$, and $\mathrm{H} = 11$. The curve line is the parabola drawn through all the points r, s, t, found by plotting the lines $\mathrm{R}r = \sqrt{2gh}$, $\mathrm{S}s = \sqrt{2g\mathrm{H}}$, &c., and which, therefore (as in Fig. 16), represent the space that would be traversed, in the horizontal, in one second by a particle of water issuing at that depth.

Hence, the area of the parabolic zone, $\mathrm{R}rst\mathrm{T}$ multiplied into l, the width of the sluice (which is shown in elevation at the right-hand side), will give the true volume of water discharged in one second at that depth.

The simple formula, § 13, gives the volume as the rectangle RLNT multiplied into the same l, LN being drawn vertically through s, for $\mathrm{S}s$ represents the velocity at the mean depth $\mathrm{H} = H + h \div 2$.

If through the point s we draw a tangent, or any right line, terminated by the parallels RL and TN, produced if necessary, then the trapezium so formed would be equal to the rectangle RLNT, and the more nearly the curve *rst* approaches a right line, the more nearly identical are the results of calculating the discharge by the two formulæ. Now, as the curve *rst* is always concave towards the vertical AT, the true formula gives a less result than that in § 13. It may be observed also, that as we descend, the portion of the curve between the equidistant parallels is more nearly rectilinear, as at B, C, and s successively, and therefore the results also gradually approach equality, and Ss is more nearly the arithmetic mean between Rr and Tt.

TABLE *showing the Error of calculating the Discharge by the Formula*, § 13.

1			2	3	4 Difference of 2 and 3.	5 Col. 4 × 0.62 × 4.	6 Col. 4 as per cent. of Col. 2.
h	H	H	$\sqrt{2g}.\sqrt{\mathrm{H}}.(H-h)$	$\sqrt{2g}.\frac{2}{3}(H\sqrt{H}-h\sqrt{h})$			
0	1	2	16.000	15.085	0.915	2.270	5.7
2	3	4	27.712	27.581	0.131	0.325	0.475
4	5	6	35.777	35.717	0.060	0.149	0.17
6	7	8	42.332	42.295	0.037	0.092	0.087
8	9	10	48.000	47.975	0.025	0.062	0.052
10	11	12	53.066	53.047	0.019	0.047	0.036
12	13	14	57.689	57.674	0.015	0.037	0.026
14	15	16	61.968	61.956	0.012	0.030	0.019
16	17	18	65.970	65.960	0.010	0.025	0.015
18	19	20	69.742	69.734	0.008	0.020	0.0116

The first column gives the successive depths below the surface; the second, the corresponding area of the rectangle Ss × RT; the third, that of the parabolic zone between the same parallels; the fourth, the difference between these two *areas;* and the fifth, the difference of the *volumes*, given by the two formula, in one second through the opening 2 ft. by 4 ft., the coefficient of contraction being supposed 0.62; in the sixth the per-centage of error by using the simple formula is shown.

EXAMPLE.—A reservoir has on one side a rectangular orifice, 2 ft. wide and 1.4 ft. deep; the surface of the water being maintained at a constant height above the sill of 2 ft. = H; required the height z' due to mean velocity of the flow. As $H = 2$ ft., we have $h = 2 - 1.4 = 0.6$ ft., and

$$(a) \quad . \quad . \quad . \quad z' = \frac{4}{9}\left(\frac{2\sqrt{2} - 0.6\sqrt{0.6}}{2 - 0.6}\right)^2 = 1.2664 \text{ ft.};$$

and as $(H + h) \div 2 = 1.3$ ft., the difference 0.0336 feet shows the error of the simpler formula when $H = 2$ feet; had it been 6 feet, all else being the same, then from (a) $z' = 5.29$ feet, and as $H + h \div 2 = 5.3$ feet, the error is now only 0.01 feet. The mean velocities are in the first case $\sqrt{2g \times 1.2664} = 9.0308$ ft. and $\sqrt{2g \times 1.3} = 9.15$ feet; in the latter they are 18.4556 feet and 18.475 feet.

49. *The charge must be measured from the level of still water.*—It may be proper here to observe, that during the flow of water through an orifice the surface of the water in the reservoir takes a curved form for a certain distance, and bends towards the side in which the orifice is opened; so that the vertical height of the surface above any point in the orifice, estimated beyond the place at which this curvature commences, is greater than that immediately over the orifice. The former of these heights or charges is that which must be introduced into the formula for the discharge, as will be mentioned in § 55. By overlooking this, as has been frequently done, an error is introduced into the calculation of the discharge, which is thus deficient; and in some cases, though very rarely, it may amount to a tenth part. The error diminishes as the charge increases, and, according to Poncelet, becomes insensible when the charges exceed 0.5 feet to 0.66 feet. This, however, refers to the small experimental orifices used by him. D'Aubuisson states that he measured the depression of the surface at the lock-gates of the canal of Languedoc, and found it, when both sluices were open, from 0.1312 feet to 0.1640 feet.

50. *Coefficient for the reduction of theoretic to actual discharges.*—The discharges which have been spoken of (§§ 46, 47) are the theoretic discharges. In order to reduce them to the actual discharges, it is necessary to multiply them by coefficients deduced from experiment. MM. Poncelet and Lesbros have determined them as in the Table following:—

TABLE *giving the "Coefficients" for the Reduction of the Discharge through Orifices calculated by the true formula, to the actual Discharge.*

Charge on the Centre.	HEIGHT OF THE ORIFICES.					
	Feet. 0.656	Feet. 0.328	Feet. 0.164	Feet. 0.098	Feet. 0.065	Feet. 0.032
Feet. 0.032						0.712
0.065				0.644	0.667	0.700
0.098				0.644	0.663	0.693
0.131			0.624	0.643	0.661	
0.164			0.625	0.643	0.660	
0.196		0.611	0.627	0.642		
0.262		0.612	0.628	0.640		
0.328		0.613	0.630	0.638		
0.393	0.592	0.614	0.631			
0.492	0.597	0.615	0.631			
0.656	0.599	0.616	0.631			
0.984	0.601	0.617				
1.640	0.603	0.617				
3.281	0.605					

51. The numbers given above are the true coefficients of contraction of the fluid vein, or coefficients of reduction of the theoretic to the actual discharge; for theory gives no other general formula for the flow of water through orifices than—

$$\frac{2}{3} l \times (H\sqrt{H} - h\sqrt{h}) \times \sqrt{2g}.$$

That which has been established in § 13, namely, $S \times \sqrt{2g\mathrm{H}}$, in which $\mathrm{H} = \frac{1}{2}(H + h)$ is only applicable in some particular cases, but which, no doubt, are of very frequent occurrence; in which H is three or four times greater than $H - h$ (Fig. 17). In all others it is to a certain extent erroneous; and the coefficients adapted to that formula, and which it has served to determine, are so also: those, namely, in the Table § 21, which are above the transverse line in each column. The coefficients that are below these lines, though determined by the aid of the formula, $S\sqrt{2g\mathrm{H}}$, are coincident with those derived from the general formula, and are correct. Moreover, in the equation $Q = m\,S\sqrt{2g\mathrm{H}}$, the error of the coefficient m is compensated by that of the formula, and the discharges assigned by it are sensibly identical with those of the true general expression; and as it has the advantage in simplicity, it is generally employed in every case.

52. Example.—Required the discharge through a rectangular orifice $0^{m}.30$ wide and $0^{m}.15$ high, and having a charge of only $0^{m}.05$ upon the upper edge: we have $H = 0^{m}.05 + 0^{m}.15 = 0^{m}.20$ and $l = 0^{m}.30$. The charge upon the centre is therefore $0^{m}.125$, and the corresponding coefficient, according to the above Table, is nearly 0.603,—the mean between 0.592 and 0.614. Thus the discharge will be—

$$\frac{2}{3} \times 0.603 \times 0.30 \times 4.43 \left(0.20\sqrt{0.20} - 0.05\sqrt{0.05}\right) = 0^{mmm}.0418.$$

The usual formula $Q = m\,S\sqrt{2gH}$, with $m = 0.592$, derived from the Table § 21, gives—

$$0.592 \times 0.30 \times 0.15 \times 4.43\sqrt{0.125} = 0^{mmm}.0417.$$

53. *Flow of water over waste-boards, weirs, &c.*—If a rectangular opening with horizontal base be formed at the upper part of one of the sides of a basin, the water—supposed to be maintained at a constant level—will flow out in the form of a sheet over this base or sill: such are the waste-boards of canals and reservoirs, and also weirs in rivers, which extend across their course, so that the water must, when it meets with them, rise, and flow over the crest or summit.

The surface of the water (Fig. 18), before it arrives at a waste-board, and from a point C, a short distance above it, is inclined in a curved line, CD, so that its height, immediately above the sill, is not equal to AB, but only BD.

54. If we followed the usual theory, we should at once grant that the particles which followed the curve CD had the same velocity when arrived at D as if they had fallen freely from the height AD, and that all the particles in the vertical line under it would in like manner flow out with velocities due to their heights, below the point A: we should thus find, that in respect to the issuing velocity of the fluid threads, and also in respect to their number,—which depends on the height BD,—and consequently in respect, also, of the discharge, that the case would be identical with that of a rectangular orifice closed on the upper side as far as D, and in which the water level extended without any curvature up to A; and, therefore, if we should put Q for the volume of water discharged in one second, l for the width of the opening, and H and h for the charges,—the one on the lower, the other on the upper edge,—and lastly, m for the coefficient of the reduction of the theo-

retic to the actual discharge, we should have the expression, as in § 47,—

$$Q = \frac{2}{3} \sqrt{2g} \; . \; m \; . \; l \; (H \sqrt{H} - h \sqrt{h}).$$

55. However natural it appears to proceed thus, facts show that the discharges are more exactly given by the supposition that the flow takes place as if from the whole height AB, the level of the water being supposed continued without curvature up to A. The case then would be identical with that of § 46: for when $h = 0$ we should have—

$$Q = \frac{2}{3} \sqrt{2g} \times m \times l \times H \sqrt{H} = 5.35 \; mlH \sqrt{H}.$$

The flow of water, then, over weirs or waste-boards is only a particular case of the flow by orifices in general; that, namely, in which the charge over the upper edge is equal to zero. MM. Bidone and Poncelet had already shown that this was so, and that the coefficients *m*, which suit ordinary orifices, are adapted to weirs also, when the flow is made under analogous circumstances.

We cannot, then, expect this coefficient to be constant; it varies, in fact, through a considerable range, from about 0.5 to 0.8, in circumstances, some of which will be recognised as nearly identical with those which influence the discharge through orifices. In determining, then, this coefficient, it is necessary, in the first place, to be satisfied that the velocity of approach of the water in the channel supplying the overfall be not so great as to produce any appreciable increase in the discharge, and thus vitiate the result: whether the object be experimentally to determine the coefficient in the formula, or the gauging of a water-course: for either end it will appear, §§ 56 and 66, that the transverse area of the sheet of water passing through the overfall should not exceed the fifth part of that of the channel.

And not only the *area*, but the *form* of the rectangle of the overfall, relatively to that of the transverse area of the channel, should observe certain limits, if we would expect to arrive at a true value for the coefficient. Thus, let us suppose a rectangular opening cut in a thin plank or metal plate, the vertical centre lines of the channel and overfall coinciding, as in the end view, Figs. 19 and 21. The fluid threads, in all that part of the channel not immediately opposite to the opening, converge towards its edges: the longitudinal section, Fig. 18 (transverse to the overfall) represents the contraction, so far as the sill is concerned, the water below the line EB bending upwards as it approaches the bottom line or sill B: the wood-engraving, § 62, which represents the plan of an overfall, shows the contraction that takes place at the sides, L being the width of the channel of supply, and *l* that of the overfall; and this convergence, continuing after the fluid has passed the edge, produces, as in orifices, a contraction of the section of the sheet of water which diminishes the discharge.

Now if the form of the opening be altered, the area still continuing of the required proportion to that of the channel, i. e. $\frac{1}{5}$th, so that in the two extreme cases, when, on the one hand, $L = l$, and therefore the depth of the sheet of water not more than $\frac{1}{5}$th of that of the channel, we would have the *end* contraction suppressed; and, on the other hand, if the depth of the overfall were equal to that of the channel, the length *l* being reduced accordingly, we would have the *bottom* contraction suppressed, as with orifices in Fig. 9, § 25. Moreover, in the intermediate rectangular forms of the overfall, in which both contractions come into operation, we may trace a difference of coefficients at different depths.

A floor or channel is sometimes attached either above or below the line of the sill, and vertically on either or both sides, above or below, alterations in the value of the coefficients resulting in each case. It must be evident also, that if, instead of a thin plank or metal plate, the sill and sides were of considerable thickness, and not bevilled off, the sheet of water, especially if the depth be inconsiderable, and the velocity consequently small, would adhere to the surface, and the coefficient become modified, somewhat like that of the case in § 67 when a floor is added below.

The effect of the contraction on the bottom line is evidently the same on each unit of the length; but the end contraction is constant with the same head, however the length may be increased or diminished; and therefore we cannot anticipate that *m*, § 59, p. 45, which includes the effect of both the end and bottom contractions, should be rigidly constant when *l* varies: an attempt to express the effect of each in the formula is given in § 62.

56. *Effects of the velocity of approach.*—When establishing the formulæ given above, it was assumed that the water was free from any current above that point at which the surface begins to be curved towards the sill; but frequently the water arrives at this point with an initial velo-

city; in this case we proceed as in § 30, for the case of orifices properly so called: that is, we add to the head due to the velocity in the case when the water is in repose (and which is now only $\frac{4}{9}H$, *vide* § 48), the head which would generate the velocity with which it arrives. Let u represent this velocity, and h the charge due to it; then—

$$\left(\text{since } h = \frac{u^2}{2g} = \frac{1}{64.4} \times u^2\right),$$

we have $h = 0.01553\, u^2$, $\left(\text{as } \frac{1}{64.4} = 0.01553\right)$,

representing the additional head in feet; and we shall have, for the actual velocity of the issuing water,—

$$\sqrt{2g\left(\frac{4}{9}H + h\right)} = \sqrt{2g\left(\frac{4}{9}H + 0.01553\, u^2\right)},$$

which may be reduced to—

$$5.35\sqrt{H + 0.035\, u^2};$$

and consequently,—

$$Q = 5.35 \times m \times l \times H \times \sqrt{H + 0.035\, u^2}.$$

The quantity u represents the *mean* velocity of the section of the water approaching the edge. The exact determination of it is, perhaps, impossible; but as its value will be very little different from that of the *surface* (which can be readily determined by methods to be subsequently pointed out), we may, therefore, grant the truth of the equation, and modify the value of the coefficient that is to be determined experimentally. If, then, we put m' for this new coefficient, and w for the velocity of the surface of the water, we shall have—

$$Q = 5.35\, m' \times l \times H\sqrt{H + 0.035\, w^2}.$$

57. We shall now put the formula § 55 to the test of experiment. The expression for the discharge contains two quantities, which vary in the experiment,—the *width* of the overfall; and a function of the velocity, that is, of the *charge*. Now, in order that the formula be well founded, it is necessary that the discharge be exactly proportional to each of these; then, and then only, will the coefficients be constant. The degree in which they are so in the experiments will thus be a test of their being well founded in truth. The numerous experi-

ments of M. Castel, engineer to the water-works of Toulouse, are quoted by D'Aubuisson, and a description of his apparatus is also given.

At one extremity of a wooden canal or trough, Figs. 19 and 21, 19.68 ft. long, and 2.427 ft. wide, and 1.804 ft. deep, he received the supply of water, and at the other he had the power of placing different thin plates of copper, in which the overfalls were cut out; their width was increased gradually from 0.0328 feet up to 2.427 feet; the sill being maintained always 0.557 feet above the bottom of the canal. The water discharged was received at pleasure, and for any required length of time, in a cistern lined with zinc, the capacity of which was 113 cubic feet: this was the gauge basin; it was marked with the greatest care by a vertical scale. The time that the water might take to reach any height was taken by a chronometer marking quarter seconds.

58. The charges or heights of the water above the sill of the overfall were successively increased from 0.0984 feet up to 0.328 feet, and even to 0.787 feet for the narrower overfalls. The most important, and at the same time the most difficult, point in the experiments, was to measure the charges exactly. In order to accomplish this, M. Castel placed over the centre of the canal, and parallel to its length (Fig. 20), a bar truly horizontal, which carried at every $0^{m}.05 = 0.164$ feet, ten vertical brass rods, divided into millimetres, pointed, and having a power of sliding up or down: on the edge of the guides was a vernier, which subdivided the rods into tenths of a millimetre.

When an experiment was made, the requisite quantity of water was admitted into the canal; and the regime or regulation for due supply of this being carefully attended to, he let down the rods, and put the points as accurately as possible in contact with the surface of the current. Subtracting then their several readings from the vertical distance between the horizontal bar and the sill, he obtained the ordinates of the curve described by the particles of the fluid as they advanced to the centre of the overfall. The ordinates increased in proportion to their distance from it; and thus the greatest ordinate, or the true charge, H, was obtained; the smallest—that, namely, which was immediately above the sill—was $H - h$, or the thickness of the sheet of water at the moment of its passage over the edge of the waste-board.

59. After having to some extent exhausted all the observations which could be made with the canal of $0^{m}.74 = 2.427$

feet in width, M. Castel entered upon those in which he used a width of only $0^{m}.361 = 1.180$ feet (Fig. 21), narrowing up the former by two partitions in plank, but whose length was only 7.347 feet, the total length of trough or channel being 19.68 feet. At the entrance A of this small canal, which was placed in the centre of the larger, there was formed, with the large discharges, a minute fall, which would have introduced some slight modifications into the results obtained, if the partitions had been extended up to the extremity of the larger canal.

Upon both the one and the other M. Castel made a long series of experiments. Each observation was repeated once or twice: altogether he made 494. In every case, the values of Q, of l, and H, were determined directly by measurement at each experiment; and from them it was easy to deduce the value of the coefficient m from the formula—

$$Q = 5.35\,mlH\sqrt{H} \text{ for } m = \frac{Q}{5.35 \times l \times H \times \sqrt{H}}.$$

60. The mean values obtained from each charge and each width of overfall are arranged in the two following Tables. No observations have been made in those cases in which there are blanks.

TABLE *showing the "Coefficients" for Overfalls, in Castel's Experiments, as depending on the Charge and on the Length.*

Charge upon the Sill.	Canal—2.427 feet wide. Coefficients, the length of Overfall being											
	Feet. 2.42	Feet. 2.23	Feet. 1.96	Feet. 1.64	Feet. 1.31	Feet. 0.98	Feet. 0.65	Feet. 0.32	Feet. 0.16	Feet. 0.09	Feet. 0.06	Feet. 0.03
Feet.												
0.78								0.595	0.615		0.639	
0.72								0.594	0.614		0.639	
0.65							0.596	0.594	0.614	0.629	0.640	0.670
0.59							0.595	0.594	0.613	0.628	0.641	0.672
0.52							0.595	0.592	0.613	0.628	0.642	0.674
0.45						0.603	0.593	0.592	0.612	0.628	0.643	0.675
0.39					0.621	0.604	0.592	0.591	0.612	0.628	0.645	0.678
0.32		0.657	0.644	0.631	0.621	0.604	0.593	0.591	0.612	0.627	0.648	0.687
*0.26	0.662	0.656	0.644	0.632	0.620	0.606	0.595	0.592	0.612	0.627	0.652	0.698
0.19	0.662	0.656	0.645	0.632	0.622	0.610	0.604	0.595	0.612	0.628	0.658	0.713
0.16	0.662	0.656	0.644	0.633	0.626	0.616	0.611	0.597	0.613	0.629	0.663	
0.13	0.662	0.656	0.645	0.636	0.632	0.623	0.619	0.604	0.614		0.669	
0.09	0.663	0.660	0.651	0.642	0.646	0.631	0.624	0.618				

Charge upon the Sill.	CANAL—1.180 feet wide. COEFFICIENTS, the length of Overfall being									
	Feet. 1.180	Feet. 0.984	Feet. 0.656	Feet. 0.328	Feet. 0.301	Feet. 0.259	Feet. 0.164	Feet. 0.098	Feet. 0.065	Feet. 0.032
Feet. 0.787				0.619			0.624	0.629	0.647	0.666
0.721				0.615	0.613	0.617	0.620	0.627	0.646	
0.656				0.611	0.608	0.614	0.618	0.626	0.645	0.667
0.590			0.633	0.608	0.606	0.610	0.616	0.626	0.644	
0.524			0.628	0.605	0.603	0.608	0.615	0.625	0.644	0.668
0.459		0.678	0.624	0.603	0.601	0.605	0.614	0.624	0.644	
0.393	0.700	0.666	0.620	0.600	0.599	0.603	0.614	0.623	0.646	0.674
0.328	0.684	0.656	0.617	0.598	0.598	0.600	0.614	0.624	0.648	
*0.262	0.672	0.652	0.616	0.599	0.597	0.599	0.613	0.624	0.654	
0.196	0.669	0.652	0.617	0.600	0.597	0.600	0.613	0.626		
0.164	0.667	0.653	0.620	0.605	0.604		0.614			
0.131	0.668	0.653	0.624	0.613	0.611		0.613			
0.098	0.670	0.665	0.632	0.628	0.625					

61. Let us next examine the same simple and common formula, $Q = 5.35\, l \times H\sqrt{H}$; as to the proportionality of the discharges Q, to the function of the charge, $H\sqrt{H}$. For this purpose let us take the twenty-two series of discharges obtained, each with the same width, but with different charges; and reducing the discharges of each series to that which they would have been if one of them—that, for instance, obtained under a charge of 0.262 feet—had been taken as unity.

TABLE *showing that the Discharges are proportional to* $H\sqrt{H} = H^{1.5}$

Charges H.	Series of Discharges.			Series of	
	1.	2.	3.	$H\sqrt{H}$	$H\sqrt{H} - h\sqrt{h}$
Feet. 0.656		3.96	3.98	3.95	4.01
0.590		3.38	3.39	3.38	3.42
0.524		2.83	2.84	2.83	2.87
0.459		2.31	2.32	2.31	2.34
0.393		1.83	1.84	1.84	1.86
0.328	1.40	1.39	1.40	1.40	1.41
*0.262	1.00	1.00	1.00	1.00	1.00
0.196	0.650	0.652	0.650	0.650	0.643
0.164	0.494	0.498	0.495	0.494	0.486
0.131	0.354	0.381	0.354	0.354	0.345

In like manner reducing the series of values of $H\sqrt{H}$, and placing side by side these several series: we have the above Table. Under the head of "Discharges" are three columns; the two first having been given by the canal, 2.427 feet in width, with overfalls 1.968 feet, and 0.328 feet in length; the third being from the canal of half the above width, 1.180 feet, with an overfall 0.164 feet long.

It results from a comparison of the twenty-two series of discharges with one another, and with the series derived from the function $H\sqrt{H}$—that the numbers in the same horizontal line are very nearly identical; and this, notwithstanding so great a difference in the charges, and in the length of overfall, both in the canal 2.427 ft. wide, and that of one-half this width: thus the proportionality of the discharges to the function $H\sqrt{H}$ may be considered as established. In the highest charge, and especially in column 3 of Discharges, the velocity of approach seems to have increased the relative discharge: as is always observed when the area of the overfall is more than ⅕th that of the channel: under column 2, with the smallest charge, the relative number is considerably in excess: irregularities of this kind caution us to avoid these small heads in practical gaugings.

Experiments to determine the index of H (which in the expression § 55 is taken at 1.5) are given by J. B. Francis ("Lowell Experiments," pp. 88, 95: Boston, U.S., 1855). The scale of these experiments was extremely large, and both the mechanism and method of observing evince the greatest care and accuracy. A large *constant* volume of water, not necessarily known in quantity, was discharged through a series of overfalls of different lengths, the extremes being (very nearly) 17 and 3.5 feet, the respective depths in one series being 0.3576 and 1.0447 feet. The average value of the index, resulting from seventy-one experiments, is 1.47. Although the volume of water discharged was not a necessary datum in calculating this index, it may be interesting to state, as showing the large scale at which this author operated, that the discharge due to the lengths and depths above mentioned is upwards of 12.5 cb. feet per second. Castel's largest experiments § 60, with a length of 2.23 ft. and depth of 0.32, gives about 1.4 cb. feet per second.

62. *Ratio of the discharges to the width of overfalls.*—The coincidence of the formula with the experiments is not so close when it is the length of the overfall which we vary. The dis-

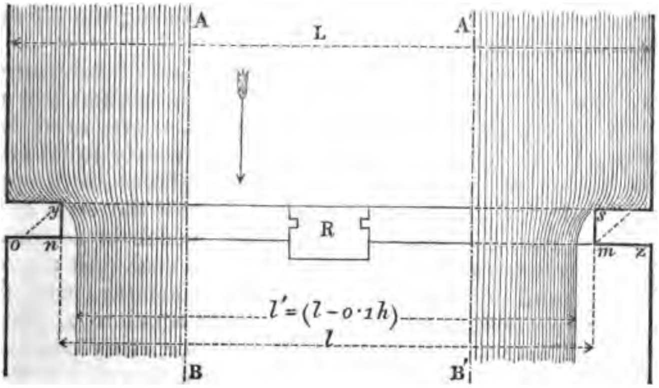
charges, considered with reference to the variable *l*, do not follow the exact ratio of the widths, however natural it would appear to suppose beforehand that it would be so. Starting from an overfall the full width of the channel, they diminish with its decreasing width, but considerably faster than it up to a certain point, beyond which, on the other hand, they diminish less rapidly than the width. The columns of the Table appended serve to fix the ideas upon this point.

TABLE *showing the Ratio of the Discharges to the width of Overfall.*

CANAL.			
2.427 Feet.		1.180 Feet.	
Width.	Discharge.	Width.	Discharge.
1000	1000	1000	1000
919	911		
811	788	831	807
676	645		
540	507	554	507
405	371		
270	243	277	246
135	121	138	125
68	62		
40	40		
27	27		
13	14		

For the canal of 2.427 feet in width, we have twelve widths, which are to one another in the ratio of the numbers in the first column: in the second we have the order in which the corresponding discharges decrease, and which were obtained under charges varying from 0.196 feet to 0.328 feet. In the case of the canal 1.180 feet wide, in which ten widths were experimented upon, those only are stated which were nearly the counterparts of the numbers in the other. The series of these ratios point out the fact, that in the two canals the discharges follow the same law in reference to the widths of the overfalls; but it is to the relative widths in their respective channels, and not to the absolute widths.

The wood-cut is intended to illustrate the effect of the end contraction that takes place at overfalls and weirs; L is the width of the channel of supply, the water in which is flowing in the direction of the arrow, and l the length of the overfall, at the place where the channel has been contracted by the projecting edges o, n, y, and z, m, s. The fluid threads are supposed to flow in lines parallel to the sides, but as they approach the narrower opening of the overfall, the length of which is l, those that move nearest the sides bend out of their course, converging towards its vertical edges throughout the whole depth; and this convergence, as in the *vena contracta* (§§ 15, 16), continues after the water has passed the points y and s, effectively shortening the length of the sheet of water passing over, which from l becomes l', and thus diminishes the discharge. The convergence of the several lines is, however, less and less as their distance from the sides of the channel increases, so that at a short distance the direction of the fluid is quite unaltered; we may suppose the dotted lines AB and A'B' to include between them the portion of the channel and of the sheet of water unaffected by the convergence.



It has almost always been assumed in formulæ for the discharge of water over weirs, that the quantity is directly proportional to the length, the depth being the same. As this end contraction is a *constant* quantity, when the length is considerable in proportion to the depth, this supposition is not well founded: a double length $2l$ giving more than a double discharge, and a length of $\frac{1}{2}l$ giving less than half the discharge, because the same constant quantity is to be deducted from each to give the effective length when the head is invariable, and in like manner of other multiples of l. It is evident that the amount of this diminution of length from end contraction must increase with the depth; the exact law of its variation is not known, but experiments have pointed out that it is very nearly in direct proportion to the depth, which is of great importance in simplicity, when we attempt to introduce it into a formula for practical application. Thus, though neither this assumption, nor that of the index of H being constant, are perfectly correct—we find that they each lead to a result agreeing very closely with experiment.

The formula proposed by J. B. Francis (Lowell Experiments, p. 86), is

$$Q = C(l - bnh)\,h^{a},$$

The quantity C, which this author calls the coefficient, stands for $\frac{2}{3}\sqrt{2g} \times m$ in the formula § 55. (Other authors make the coefficient represent $\frac{2}{3}m$; as Morin, 2nd edit., in giving the Tables of Lesbros experiments. No error, but a great want of uniformity only, is caused in so doing; it requires care in comparing, and labour in reducing the tables of different authors.)

By the letter n is represented the number of end contractions, for we frequently find at weirs across streams (though not in experimental overfalls) that vertical posts, as at R in the wood-cut, are placed along the crest, at nearly equal intervals, having vertical grooves, into which planks are dropped horizontally; these, being staunched against the crest of the masonry, raise the surface of the water, which in seasons of drought is necessary for many purposes; at each post the convergence and consequent contraction are identical with that at the two ends; at the least, then, n is equal to 2, that is, when the length is uninterrupted, and for every post it increases by 2: the constant multiplier or fraction of h is represented by b, and is such that $bh = \frac{l - l'}{2}$.

The coefficient b and index a are in one point distinct from m, inasmuch as they can be determined by experiments in which the actual discharge is not known in any particular case.

It is evident that this formula cannot be of general application, for if in the factor $l - bnh$, which represents the effective length, we should have $l = bnh$; then this effective length becomes 0, and the formula would give 0 for the discharge, which is absurd; similarly, if $bnh < l$, the discharge would be negative. In weirs of very short length in proportion to the depth, the effect of the end contraction cannot be considered independent of the length. It is found by experiment that, when the length equals or exceeds three times the depth, the formula applies; but in lengths less than this in proportion to the depth, it cannot be used with safety; the error increasing as the relative length of the weir diminishes. The end convergence influences the discharge to a certain distance from the end of the weir, as, suppose, from y to the line AB; or from s to A′B′, if the whole length of the weir is greater than twice this distance, the effect of the end contraction is independent of the length; but if the length is less than twice this distance, the whole breadth of the stream is affected in its flow by the end contractions, and consequently the proposed formula would not apply. In practical construction it is nearly always possible so to proportion the weir, that the length may not be less than three times the depth upon it. In cases where the length of the weir is equal to the width of the channel, that is, $l = L$, there is no end contraction, so that we have $L = l = l'$, and the proportion of the length to the depth is not material.

The formula proposed by Francis, given above, deducts from the length l of the overfall a quantity presumed to represent the end contractions, and gives the value of l' the effective length. It is, giving b and a their numerical values,

$$Q = C(l - 0.05nh)h^{1.47}.$$

At the end of his second set of experiments Francis gives

$$Q = C(l - 0.1nh)h^{\frac{3}{2}}.$$

The use of the fractional index 1.47, not being so convenient, it has been increased 2 per cent., and the quantity deducted from l doubled, giving results agreeing most nearly with all his experiments. If $n = 2$, that is, if no vertical posts, as R, are fixed on the crest, then the quantities to be deducted from l are $0.1h$ in the former, and $0.2h$ in the latter formula.

63. *As to the coefficients.*—Since the discharges are for the same widths sensibly proportional to $H\sqrt{H}$, when we omit extreme cases,—the coefficients ought to be very nearly constant, and we find they are so in the Table § 60. In strictness, when we take the coefficients of some one vertical column of the Table, we see them—commencing with the higher

charges—decrease by very small degrees, in most of the experiments, down to a certain charge, beyond which they increase rapidly; thus we shall have at this particular charge, which is generally about 0.328 feet, a minimum. And since, the charges remaining constant, the discharges decrease at first more rapidly than the widths of the overfalls, and afterwards less so, it follows that under the same constant charge—the widths, commencing at a first width equal to that of the channel itself, being diminished—the coefficients decrease down to a certain point, beyond which they augment. Thus, here also there is a minimum; and it occurs when the width of the overfall is about the fourth part of that of the supplying channel; and so in both the horizontal and vertical lines of the Tables of coefficients there is a minimum; we have, therefore, a common minimum. In its immediate neighbourhood, and for a certain distance, the variations are small; the coefficients in that part differ very little from one another, and may be regarded as constant. But in the other parts of the Table the differences rise to be considerable; they exceed an eighth, or 12 per cent.; so that the discharge by overfalls cannot be given exactly with a constant numerical coefficient, *m*, in an expression of the form $H \sqrt{H}$. In practice, then, we would require the aid of very extended tables of coefficients, the preparation of which would demand many hundred experiments. However, the study of the direction in which the coefficients tend gives us the means of abridging this vast labour, and of determining a few simple rules suitable to the different cases which commonly present themselves.

64. *Coefficients and formulæ to be employed.*—We have seen (§ 61) that the expression $lH\sqrt{H}$ should not be employed—on the one hand, when the charges were below 0.196 ft.; and on the other, when the transverse area—that is, the depth multiplied by the width of the overfall—exceeds the fifth part of the area of the section of the water in the supplying channel, the initial velocity becoming then considerable. Between these limits, the expression above given can be employed with a coefficient,—variable, it is true, but only varying with the width of the overfall.

Reckoning from a width equal to that of the canal itself, the coefficients diminish with the width of the overfall until it has reached the fourth part of that of the channel, and then they increase, although the widths are still decreasing; and what is very remarkable is, that the diminution of the coefficients follows the relative widths of the overfall to the canal, whilst the augmentation, which occurs afterwards, depends only on the absolute width. We have, consequently, four cases to distinguish relative to the coefficient to be used:—

First. In the neighbourhood of that common minimum we have mentioned, the variations of the coefficients are very trifling. According to the experiments made by Castel, from a width of overfall nearly equal to the third of that of the canal, which is supposed to exceed 0.984 feet, down to an absolute width of 0.164 feet, the coefficients do not vary more than from 0.59 to 0.61. Taking the mean, and remarking that—

$$5.35\left(=\sqrt{2g}\times\frac{2}{3}\right)\times 0.60=3.21,$$

we shall have between the limits indicated above—

$$Q=3.21\,lH\sqrt{H}.$$

This formula furnishes the best mode of gauging small streams of water, as in the Examples appended.

Secondly. When the width of the overfall is at its maximum—that is, equal to that of the canal, extending from bank to bank, and is then precisely similar to a weir, properly so called,—the coefficients present a remarkable steadiness under the different heads. M. Castel, in his experiments upon the canal of 2.427 feet, with an overfall or barrier of a height equal to 0.557 feet, had no difference in the coefficients obtained with charges which themselves varied from 0.098 feet to 2.262 feet, § 60. With an overfall of 0.738 feet high, the coefficients have only ranged from 0.664 to 0.666 for charges of 0.101 feet up to 0.242 feet: at the mean he had 0.665. And since—

$$5.35\times 0.665=3.55775,$$

we have, putting L for the width of the canal or length of the barrier—

$$Q=3.558\,LH\sqrt{H}.$$

This formula will be used with advantage in certain cases, even in large water-courses, and with charges of 0.131 feet and 0.098 feet; but to insure certainty in its use, it is necessary that the charge be less than the third of the height of the barrier.

Thirdly. For widths of overfall comprised between that of the channel itself and the fourth part of the same, the coefficient of the expression $5.35\, lH\sqrt{H}$ will vary with the relative width,—that is to say, with the ratio of the width of the overfall to that of the canal of supply, and is given by the columns of this Table:—

TABLE *showing the Variation of the "Coefficient" with the relative width of the Overfall in the Formula* § 55.

Relative Widths.	Coefficient for Canal of	
	2.427 Feet.	1.180 Feet
1.00	0.662	0.667
0.90	0.656	0.659
0.80	0.644	0.648
0.70	0.635	0.635
0.60	0.626	0.623
0.50	0.617	0.613
0.40	0.607	0.609
0.30	0.598	0.600
0.25	0.595	0.598

They have been formed by taking proportionals to the coefficients deduced from direct experiments, as given in § 60, a method which cannot here lead to any error. The coefficients determined by each canal have been given separately, in order to show that coefficients sensibly the same correspond to the same relative width, although the actual value of the widths was in one canal nearly double of that of the other, affording a proof that above 0.25, or the fourth of the width of the channel, the coefficients depend on the relative, and not on the absolute width of the overfall; taking relative widths of 0.8, 0.6, and 0.25, and the mean coefficients, we have

$$Q = 5.35 \times \begin{Bmatrix} 0.596 \\ 0.624 \\ 0.645 \end{Bmatrix} lH\sqrt{H} = \begin{Bmatrix} 3.188 \\ 3.338 \\ 3.450 \end{Bmatrix} lH\sqrt{H}.$$

Fourthly. It is quite otherwise when this width falls below that of the fourth of the supplying canal: then, and when at the same time it is absolutely less than 0.262 feet or 0.196 feet, that of the canal has no further effect, and every particular width has its own coefficient. Thus in the canal of 1.180 feet, as well as in that of 2.427 ft.,

the widths, . . 0.164 ft, 0.098 ft., 0.065 ft, 0.033 ft.,
have given, . . 0.61, 0.63, 0.65, 0.67,

as the respective coefficients in both canals.

65. *Observations on formula of* § 54.—Having thus given in detail all that has reference to the simplest of the formulæ of discharge for overfalls, let us consider the two others, and first, that of—

$$Q = 5.35\, ml\, (H\sqrt{H} - h\sqrt{h}),$$

in which h represents the quantity AD (Fig. 18), the surface of the fluid having become curved before its arrival at the overfall. A slight inspection of the last column of the Table given in § 61 shows, that although the series of values of $H\sqrt{H} - h\sqrt{h}$ does not differ much from the series of the corresponding discharges, yet that it follows them less closely than those of $H\sqrt{H}$. Thus in this important point, the second formula is not so well established as the first. It is, moreover, of much more difficult application, containing an additional term, and one whose exact determination is a matter of very great difficulty.

66. *Observations on the formula in* § 56.—This formula, which involves a term that is a function of the velocity with which the water flowing in the canal arrives at the overfall, is well founded and of practical utility.

It is evident that in the case of a high velocity, in which the flow takes place both from the charge H and from an *initial velocity*, w, taken at the surface, it is necessary to add to the charge a term depending on this velocity, which leads to the equation, § 56,

$$Q = 5.35\, m'lH\sqrt{H + 0.035\, w^2}.$$

The experiments of M. Castel give the values of m' the coefficient. In these experiments the velocity w of the surface of the water in the canal has not been actually measured, but it can be determined from the mean velocity, which is equal to the discharge Q, divided by the section of the running water in the channel of supply, which in this case is $L \times (H + a)$, representing by L the width of the channel of supply, and by a the height of the sill of the overfall above the bottom of the channel; and as

$$v \times \{L\,(H + a)\} = Q \therefore v = \frac{Q}{L\,(H + a)}.$$

It will hereafter be shown that the velocity of water at the surface is, on an average, a fourth part higher than the mean velocity; so that we have—

$$w = 1.25 \times \frac{Q}{L(H+a)}.$$

Even with this value of w—which is the highest we may assume—the coefficient m' differs from the coefficient m in the common formula only when the velocity in the channel is great enough to occasion the term $0.035\ w^2$—which is that which makes the difference between the two formulæ—to have a value comparable to that of H. As it will be in most cases very small, and as it is under the radical sign, it will only influence the value of m' by half its amount relatively to H; for example, if it is the 2, 4, or 6 hundredths of H, the coefficients, *ceteris paribus*, will only differ by the 1, 2, or 3 hundredths. In these three cases the section of the sheet of water at the overfall, or $l \times H$, is found to be respectively equal to 0.1724, 0.244, and 0.3, of the section of the canal of supply, or of $L\ (H+a)$, whence may be deduced the conclusion of which we have already made use (§§ 61, 64), that when the former of these two sections is less than the $\frac{1}{5}$th part of the latter, the coefficients m and m' will be the same within a hundredth part nearly. Such has been the case with the overfalls used by M. Castel, whilst their width has been less than the half of that of the channel. When it was greater the term $0.035\ w^2$ has had more effect, and the differences became larger. But the use of this term is far from having brought to equality the coefficients m and m' for various widths of overfall: it has not reduced even by half the differences which occur in the values of m; and neither the expression—

$$Q = 5.35\ m'lH\sqrt{H + 0.035\ w^2},$$

nor that of

$$Q = 5.35\ mlH\sqrt{H},$$

can be employed with a constant coefficient, except in the case of a width of overfall equal to that of the canal of supply.

In order to obtain the coefficient in this case, M. Castel dammed up the canal of 2.427 feet by means of barriers of copper, whose height was decreased successively from 0.738 feet to 0.104 feet; and he has thus obtained the coefficients in this Table:—

TABLE *showing the Variation of the Coefficient in the Formula* § 56, *due to circumstances affecting the Velocity of Approach.*

Height of the Dam.	Coefficients m', the Charge being			
	0.26 Feet.	0.19 Feet.	0.16 Feet.	0.13 Feet.
Feet.				
0.738	0.651	0.655	0.657	0.660
0.557	0.640	0.647	0.650	0.654
0.426	0.650	0.649	0.652	0.656
0.305	0.635	0.642	0.646	0.650
0.246	0.647	0.652	0.655	0.660
0.134	0.667	0.664	0.665	0.668
0.104	0.676	0.676	0.676	0.680

Those of the five first barriers are nearly the same; and although they do not present the same regularity which we had in ordinary overfalls, we may assume 0.650 as the mean. As to the last two barriers of 0.134 feet and 0.104 feet, they are in a distinct class. They were very low, and the charges very much exceeded their height; so that the case was nearly as much one of water flowing in an ordinary channel, as of it when passing over a waste-board. It may be remarked, moreever, that the near equality between the coefficients for one and the same barrier speaks strongly in confirmation of the formula which has determined them.

The experiments upon the canal of 1.180 ft., with its barrier of 0.557 ft., have indicated coefficients whose mean was 0.654. Taking, then, the mean between this and 0.650, that is, 0.652, and observing that $5.35 \times 0.652 = 3.488$, we shall have finally—

$$Q = 3.488\,LH\sqrt{H + 0.035\,w^2}.$$

The velocity w is to be determined by direct observation.

67. *Overfalls, with channels attached.*—We frequently find channels are adapted to overfalls: they may be considered as the prolongation of their horizontal and vertical edges. The water discharged is now confined, and suffers a resistance from the friction of the bottom and sides, which retards the motion, and this retardation, re-acting on the water which arrives at the overfall, diminishes the discharge. The following expe-

riments by MM. Poncelet and Lesbros exhibit the effects of this resistance. The additional channel was always $3^m = 9.84$ feet long, and of the same width as the overfall, $0^m.20 = 0.656$ feet, and adjusted so as to be horizontal:—

TABLE *showing the Effect upon the "Coefficients" produced by Channels added externally to an Overfall.*

Charge.	Coefficient.		Loss per cent.
	Without added Channel.	With added Channel.	
Feet.			
0.675	0.582	0.479	18
0.475	0.590	0.471	20
0.337	0.591	0.457	23
0.196	0.599	0.425	29
0.147	0.609	0.407	33
0.091	0.622	0.340	45

The amount of diminution in the discharge from *overfalls* with the channel attached has therefore been so much the less, as the charge has been higher. From this we may infer, that with charges of 3 or 4 feet and greater, such as are often in operation at the head of large feeders and water-courses, the diminution of discharge due to the resistance of the sides of the channel is inconsiderable. With *orifices* the same experimenters arrived at results analogous to those of overfalls. They applied the additional channel, 9.84 feet long by 0.656 feet wide, mentioned above, to the exterior of the orifices from which had been derived the Table § 21, p. 18, where it is mentioned that the orifices were all 0.656 feet wide; and from numerous experiments it was deduced that when the charges, measured from the centre of the orifice, were from 2 to 2½ times greater than the height of the orifice itself, the channel attached had no decided influence upon the discharge: it was the same in amount whether this was or was not present; but with very small charges it diminished the charge even a fourth or more.

Further investigations as to the effect of inclining the channel were undertaken. When the slope was 1 in 100, or 34′, the coefficients were found the same as when the channel was horizontal, but at 1 in 10, or 5° 44′, they were increased 3 or

4 per cent. Castel also experimented on overfalls, and on his canal of supply, 2.247 feet wide, he placed an overfall 0.656 ft. wide, with a channel attached 0.669 ft. long, inclined 1 in 13.3 or 4° 18′,—

TABLE *showing the result of Castel's Experiments on Channels added externally to Overfalls.*

Charge.	Coefficient.
Feet.	
0.364	0.526
0.311	0.527
0.249	0.527
0.196	0.528
0.164	0.530

The coefficients were obtained with the formula,

$$Q = 5.35\, mlH\sqrt{H}.$$

They vary very slightly, although the charges have more than doubled. The mean is 0.527, and would probably have been 0.53 if the inclination had been 1 in 12, which is common in practice. With the simple overfall the coefficient was 0.60; so that the additional channel diminished the discharge about 12 per cent.

68. A particular case yet remains to be considered—namely, that of the *demi-deversoirs* or *devèrsoirs incomplets*, as Dubuat has called them, or drowned weirs in English writers, so called when the tail water has risen above the level of the sill. Dubuat has divided the height of the water above the sill into two parts (Fig. 22), Ab and bC. In the former, the flow takes place as in an ordinary overfall, in which Ab ($=H$) is the charge; so that the volume of water discharged (§ 66) is expressed by $Q = 3.488\, lH\sqrt{H + 0.035\, w^2}$. In the second portion it may be assumed that the discharge is the same as in a rectangular orifice, of which bC is the height, and the charge equal to the difference of level Ab between the upper and lower surface of the water. bC is the height of this latter surface above the sill of the overfall, and if we call the height bD of the surface above the bottom of the canal, a, and the height CD of the sill above the same point, b, it will be equal to $a - b$. To the charge Ab or H is to be added, as in the

case of closed orifices (§ 32), the height due to the velocity u of the water in the canal, and the velocity of issue will be—

$$\sqrt{2g\,(H+0.01553\,u^2)}=\sqrt{2g\,(H+0.01\,w^2)},$$

since $w=1.25\,u$ (§ 66). Thus we shall have for the discharge of this part (§ 29)—

$$Q=4.96\,l.(a-b)\sqrt{H+0.01\,w^2},$$

adding the two partial discharges, and putting Q for the total discharge—

$$Q=3.488\,lH\sqrt{H+0.035\,w^2}+4.96\,l\,(a-b)\sqrt{H+0.01\,w^2}.$$

69. *Arrangements preliminary to gauging.*—Weirs constructed completely across the bed of a river may sometimes give the means of measuring the discharge; but it is in such cases necessary that the crown of the weir should have a well-defined edge, so that the water which flows over it may fall freely, and without meeting any check from the reaction of the body of water already passed over. (*Vide* § 67.) It is but seldom that they are so constructed; however, we may, without great expense, adapt a weir with the usual rounded crest to purposes of gauging by raising on the crest some temporary structure that shall have the necessary well-defined edge, and of a sufficient height, so that the discharge may not suffer from any such reaction.

We must so regulate the length of this apparatus that the depth of the water, H, flowing over may be greater absolutely than 0.1968 feet, § 61, but less relatively than the fourth part of the depth of the river as it approaches the weir. In such cases the discharge will be given by the formula $Q=3.558\,LH\sqrt{H}$ (§ 64), L being the length of the temporary crest or edge. In case H should exceed the fourth part of the depth of the current of the stream, we must use the formula in § 66, $Q=3.488\,LH\sqrt{H+0.035\,w^2}$, w being the velocity taken at the surface.

70. If the mode of gauging by weirs be but seldom applicable to larger rivers, it is, on the other hand, the most suitable for small streams and water-courses. We have two cases to consider: first, that in which the quantity of water discharged is at or under about 40 cubic feet per second. A

spot must be chosen where an overfall can readily be established, having a length, l, greater than 0.3 feet, but less than the third part of the width of the bed of the stream, and so disposed that we may have a charge, H, greater than 0.1968 ft.; all being, moreover, subject to the condition that lH be not greater than the fifth part of the section of the current immediately above the overfall; then, without fearing any error greater than the hundredth part, we may apply the formula $Q = 3.21\, lH\sqrt{H}$ (§ 64).

If, secondly, the quantity discharged should exceed 40 cubic feet per second, we must pond up the water by a dam extending from bank to bank, and at each extremity place vertical side-boards, so that the opening traversed by the water may be rectangular, the crest being truly horizontal, and using either of the formulæ mentioned above, according to the conditions specified. The two examples following will point out the manner of proceeding, and furnish an opportunity of adding some practical details, elucidating what has been laid down in § 63 to § 66.

71. Examples.—1st. Let us suppose it necessary to gauge the discharge of a small river or water-course; we must search for a part suitable for the construction of an overfall. This will probably be found at a point where the bed has become contracted, and the banks are somewhat steep, and immediately below a wide part of the stream; at such locality the width at the surface of the water is found to be 11.808 feet, and the greatest depth 2.62 feet. After a preliminary examination of the transverse section, and of the surface velocity, measured by means of some light body thrown into the current, we find, approximately, multiplying the assumed section by the velocity, that the stream is discharging nearly 35 cubic feet per second.

Since the width is 11.808 feet, we may give 3.936 feet to that of the overfall for gauging (i. e. 11.808 ÷ 3); the charge H will then be about 1.97 feet, for the formula $Q = 3.21\, lH\sqrt{H}$ gives—

$$H^{\frac{3}{2}} = \frac{Q}{3.21 \times l}, \text{ or } H = \sqrt[3]{\left(\frac{Q}{3.21 \times l}\right)^2}$$

which, for the assumed discharge, gives in this case,

$$H = \sqrt[3]{\left(\frac{35}{3.21 \times 3.936}\right)^2} = \sqrt[3]{(2.7703)^2} = 1.9725 \text{ ft.}$$

From this preliminary inspection we may construct a suitable partition of plank about 0.1 feet thick on the upper edge, and of such figure as nearly to fit the sides and bottom of the water-course. It must be carefully staunched, being sunk into the bottom and sides, and puddled on the up-stream face. From out of its upper edge we must cut an opening 3.936 ft. long and 2.132 ft. deep, so that its sill being 0.488 feet above the bottom of the bed (2.62 − 2.132), the water may flow off freely. The section of the presumed discharge (3.936 feet × 1.969 = 7.746 square feet) being not the fifth nor even the seventh part of the transverse section of the river, which exceeds 55 square feet, all the conditions for the application of the formula $Q = 3.21\, lH\sqrt{H}$ are present. Everything being duly prepared for the gauging of the water—such as, all leakage having been stopped, and the current restored to its ordinary and uniform flow, we proceed to measure H by stretching a cord across the opening, whose ends are fastened to points in the sides, marked at the level of still water (deduction being made for capillarity), and about a foot from the vertical side of the overfall. The depth of the sill below this line at the centre of the opening is carefully measured, and found to be 2.01 ft., and the length also, intended to be 3.936 feet, is found to be 3.92 feet. The discharge is, therefore, $Q = 3.21 \times 3.92 \times 2.01 \sqrt{2.01} = 35.86$ cubic feet.

72. 2nd. A question of law requires that the exact quantity of water flowing down a stream when the surface is level with the top of a certain fixed mark be determined. The gauging must evidently be effected by a dam across the course. About 170 feet above the mark a temporary dam is placed, at a part where, from its regular width and inclination, the river-bed is suitable, having, when the water is at the height above named, a breadth of 64.94 feet, and a mean depth of 4.1 feet; the overfall being a plank well squared, and 0.1312 feet thick, the upper edge being placed truly horizontal, and 0.656 feet above the bench-mark. At each extremity a vertical piece is raised, so that the length of the overfall is 63.6 feet; close to the vertical pieces two others are placed, on which a scale is drawn whose zero is the upper edge of the plank.

These arrangements being made, it is only necessary to observe when the surface of the water down stream is level with the fixed mark, and then read the height of the water upon each scale. This last has been found to be 2.34 feet.

As this height is nearly the half of that of the temporary dam $(4.1 + 0.656 = 4.756)$, we cannot apply with confidence the formula $3.558\,LH\sqrt{H}$, § 64, p. 53, and we must use—

$$3.488\,LH\sqrt{H + 0.035\,w^2},\ \S\,66.$$

To obtain the value of w, the velocity of the surface of the current on arriving at the overfall, we must take a distance of, say, 165 feet on each bank, above the point where the surface of the current begins to curve towards the overfall; mark these points, and, about 60 or 70 feet above them, place in the current some floating body of the same specific gravity as water, and mark carefully the time which it may take to flow along the 165 feet: a mean of six observations gave 48½ seconds; whence—

$$w \text{ is} = \frac{165}{48.5} = 3.4 \text{ feet per second, and } 0.0349\,w^2 = 0.40344.$$

Hence—

$$Q = 3.488 \times 63.6 \times 2.34\sqrt{2.34 + 0.40344} = 859.78 \text{ cubic feet,}$$

the formula $3.558\,LH\sqrt{H}$ would give 810.63 cubic feet. Thus we may certify, that at the given height of surface the river discharges about 850 cubic feet of water per second.

73. Experiments on weirs on a large scale have been undertaken by Mr. T. E. Blackwell. The first set, contained in the following Table, is the result of a series of 243 experiments, made on overfalls of 3, 6, and 10 feet in length, with heads from 0.0833 ft. to 1.166 ft., and with the varying circumstances of having for the overfall bar,—first, a thin plate; secondly, a plank 0.166 ft. thick, square on the top; and thirdly, a crest 3 ft. in breadth. The thin plate was a piece of iron fender plate, barely 0.0052 ft. thick; and the broad crest was an apron formed of deal boards 3 ft. long, roughly planed over, and fastened on to the outer edge of the vertical overfall plank, so as to form an uninterrupted continuation of it, the object being to approximate towards the case of well-constructed wide-crested weirs, such as may be found in actual use in rivers, &c.: the position of this planking was, in some of the experiments, horizontal, and in others sloped at 1 in 18 and 1 in 12.

FIRST SET OF EXPERIMENTS—T. E. BLACKWELL.

Length of Overfalls.	Coefficients under their respective Charges.												Mean Coefficients.
	Feet. .083	Feet. .166	Feet. 0.25	Feet. .333	Feet. .416	Feet. 0.50	Feet. .583	Feet. .666	Feet. 0.75	Feet. .833	Feet. 1.00	Feet. 1.166	
	Overfall being a Thin Plate, 0.0052 Feet.												
Feet. 3	.676	.675	.630	.616	.601	.592							.631
10	.808	.802	.642	.655	.649			.580	.529				.667
	Overfall Plank, 0.166 Feet thick.												
3	.466	.508	.562	.549	.588	.592	.616	.606	.600	.613			.570
6	.459	.561	.597	.574	.601	.607	.607	.589	.568	.538	.525	.549	.565
10	.435	.585	.568	.601	.609	.576	.571	.547	.558		.534	.534	.556
	Same Overfall with Wingboards, converging at Angle 64°.												
10	.754	.675		.655	.670								.688
	Crest 3 Feet long, sloping 1 in 12.												
3	.466	.532	.538	.454		.531	.526		.498				.507
	Crest 3 Feet, sloping 1 in 18.												
3	.544	.546	.51	.430	.516		.513	.490	.492				.508
10	.466	.495		.514		.543		.507					.505
	Crest 3 Feet, level.												
3	.451	.481	.441	.418	.478	.501	.487	.469	.475				.466
6			.492	.496			.496			.465	.466		.483
10	.381	.478			.517	.513		.468	.486	.454			.471

74. The second set consisted of a series of 70 experiments on an overfall about 10 feet in length, the width of the bar being in every case 0.166 feet.

In both, special circumstances which influenced the coefficients obtained were present; and in employing them in actual practice, a careful judgment must be formed as to their exact degree of applicability. The former were made upon a side pond or reservoir of the Kennet and Avon Canal, the area of which was 106,200 square feet; and as to these the author remarks, first, that the pond was supplied with the water, not continuously, as drawn off in the experiments, but three or four times a day, or as often as might be requisite: however, even if during an experiment no water had been admitted, the decrease of the "head" must have been inappreciable, for the largest discharge measured in any experiment was barely 444 cubic feet, which gives a fall of (444 ÷ 106200 =) 0.00418 feet over the whole surface, between the beginning and end of it, and the half of this, or 0.00209 feet deducted from the head, measured at the commencement, for the approximate mean head, could not be perceived. Secondly, at some little distance above the overfall the depth of water was reduced by a submerged course of masonry (Fig. 23), which rose to within 18 or 20 inches of the surface: the width of the approach to the overfall being about 40 feet, giving a transverse area of 60 square feet in the water approaching the overfall. Thirdly, the overfall was placed upon the outer line of the dam, in order to obtain the requisite free fall; and the depth of water immediately contiguous to it and on the dam would seem, from the section, to be only a little more than 2 feet; but the average depth between the submerged course and the dam about 3 feet. The line of the overfall, moreover, was not exactly in the direction of one of the sides of the reservoir; and from the measurements given of the head, taken at still water, and the corresponding depth of the sheet of water flowing off, as given in the following Table, it would appear that some degree of resistance opposed the motion of the water up to the overfall.

75. *Actual depth at point of discharge.*—In this Table the actual heads were measured at the outer edge of the boarded crests and edges of the overfalls. The upper line of figures gives the head, H, at still water, in feet; the other figures the heads at the outer edge of overfalls or crests, also in feet.

TABLE *showing the Heads at Still Water, and at Edge of Overfall.*

	OVERFALL.	Head at Still Water.									
		Feet. .083	Feet. .166	Feet. .25	Feet. .33	Feet. .416	Feet. .5	Feet. .583	Feet. .666	Feet. .75	Feet. .833
		Resulting Depths at Outer Edge.									
Overf. plank 2 in. thick.	3 ft. long,	.052	...	...	.25 to .224	.291	.364	...	.531	...	...
	6 ft. long,	...	...	.161	.291	.296	.361	...	...	...	...

TABLE *showing the Heads at Still Water, and at Edge of Overfall—continued.*

	OVERFALL.	Head at Still Water.									
		Feet. 0.83	Feet. .166	Feet. .25	Feet. .33	Feet. .416	Feet. .5	Feet. .583	Feet. .666	Feet. .75	Feet. .833
		Resulting Depths at Outer Edge.									
Crests of boards, 3 ft. wide.	3 ft. long level, . .	.036	.073	...	.114	.187	.198	...	...	...	...
	3 ft., slope 1 in 18, .	...	...	...	.114	.145	...	.177	...	...	...
	3 ft., slope 1 in 12, .	.0208	...	...	.125	...	...	.198	...	...	...
	6 ft. long level, .	.041	...	.093 to .104	...	...	.229	...	...	.343	...
	10 ft. long level, . .	.036	.070	...	...	.145	.218	...	.302	.291	.333
	10 ft., slope 1 in 18. .	.026	.046	...	.125	...	.187	...	.291	...	...

The general arrangement at these, the first set of experiments, would seem fairly to represent the case of the discharge of water by an overfall from a large still reservoir.

76. In the second set of experiments—those at Chew Magna—we have an area of reservoir of 5717 square feet kept constantly full by a pipe 2 feet diameter, discharging from a head of 19 feet. The distance between the mouth of this pipe and the overfall was only about 100 feet; the water must, therefore, have retained some of its velocity on approaching the overfall; and indeed, with charges above 0.417 or 0.5 feet, this was perceptible to the eye, but could not, the author states, be accurately determined, from the peculiar form of the reservoir. The results, however, show that this influence must have been considerable, and that the effect of water approaching an overfall with an initial velocity is an element which should never be disregarded. The longitudinal section, Fig. 24, as compared with that in the first set, at the Kennet and Avon Canal reservoir, seems also more favourable to the free approach of the water. The overfall had wings at an angle of 45°, well adapted for facilitating the discharge. The overfall bar was a cast-iron plate, 0.166 feet thick, with a square top. The general circumstances attending these, the second set of experi-

ments, make the discharges given by them analogous to the case of a weir in a river, or in a running stream; and in this view they have great value when carefully applied.

OVERFALL, Cast-iron Plate, 0.166 Feet thick; Length, 10 Feet.

Head in Feet.	Coefficients.	Head in Feet.	Coefficients.	Head in Feet.	Coefficients.
.083 to .073	.591	.3437	.743	.5	.749
.083 to .088	.626	.3594	.760	.5156	.748
.182 to .187	.682	.3646	.741	.5156 to .521	.747
.229	.665	.3610	.750	.5781	.772
.2435	.670	.375	.725	.639	.717
.2396	.655	.416	.780	.6666	.802
.2422	.653	.4227	.781	.66 to .734	.737
.2448	.654	.4505	.749	.7448	.750
.25 to .253	.725	.453 to .456	.751	.75	.781
.3333	.745	.4948	.728		
				Mean of all, . .	.723

77. These several sets of experiments, and those of J. B. Francis, § 80, are probably, as to length of overfall, and charges some of which exceed 0.75 feet, the largest that have been yet recorded. To compare them fully with those of D'Aubuisson, it would, perhaps be necessary to have had a greater number of widths below that of 10 feet than 6 and 3 feet, but with these only, as published, the comparison tends, in some degree, to confirm the coefficients given by D'Aubuisson in §§ 64, 66. Let us, for this comparison, recapitulate what has there been laid down. First, from the Tables, § 60, and the remarks, § 64, it appears that, with an overfall whose length is one-third or less of the channel of supply, we must use 0.60 as the multiplier or coefficient to reduce the expression—

$$\frac{2}{3} lH \sqrt{2gH},$$

so as to give the true discharge per second (p. 53). Hence,

$$Q = 3.21\, l \times H \sqrt{H}.$$

Secondly, when the overfall is of a length equal to that of the width of the channel of supply, we may use 0.665, provided also, that the head of water be less than one-third of the height of the dam above the bottom of the channel of supply, giving—

$$Q = 3.558\,LH\sqrt{H}.$$

Thirdly, we have coefficients which decrease from 0.662 to 0.595 in proportion as the length of the overfall ranges from the full width of the channel to the fourth part of it. Lastly (§ 64, p. 53, 54), we have 0.652 to be employed with overfalls whose length is the full width of the channel of supply; and when also the initial velocity of the water in this channel is represented in the formula by the quantity added to H in the factor—

$$\sqrt{H + 0.035\,w^2}, \text{ giving } Q = 3.488\,LH\sqrt{H + 0.035\,w^2}.$$

Fourthly, we have, in the case of the additional channels or broad crests, the coefficient 0.527 to be used for m in the formula—

$$Q = 5.35\,mlH\sqrt{H}, \text{ giving } Q = 2.8355\,lH\sqrt{H}.$$

78. In Mr. Blackwell's experiments we can only use for this comparison those in which the overfall was constructed of plate-iron 0.0052 feet thick; and secondly, those having the crests 3 feet broad attached below the edge; since D'Aubuisson has only recorded experiments made with a thin plate of copper for the overfall, and those which had a channel attached below the edge. Now, if we consider the 10 feet overfalls of Mr. Blackwell as being analogous to those in D'Aubuisson, in which the length is equal to the full width of the channel of supply,—which the plan (Fig. 25) would seem to justify,—we find that, taking out from Table § 73 the average coefficient, overfall of thin plate-iron and 10 feet long, we have—

According to T. E. Blackwell, ...	0.667
,, D'Aubuisson (§ 64, second case),	0.665

The average of the 3 feet overfall (less than one-third of the channel of supply), constructed of plate-iron 0.0052 feet thick, according to Blackwell, is 0.631, in the "first case" in § 64 we find 0.600 as being used by D'Aubuisson in analogous circumstances. It may, however, be observed, that the charges in Mr. Blackwell's experiments, which give 0.631, not going higher than 0.5 feet, and the coefficients decreasing up to that head, make it probable that they would have decreased much lower had the experiments been continued, and so reduced the coefficient 0.631 to a value nearer to 0.600.

Again, in the experiments with the added channels or broad crests, we find the average of Mr. Blackwell, when the crest is horizontal, to be 0.473. The average of those in D'Aubuisson, § 67, p. 58, is 0.430, but particular experiments give a closer agreement in the coefficients: for instance, if we take out that derived from the charge 0.337 feet in this last Table, the coefficient is 0.457; and in § 73, under the nearly equal charge of 0.333, we have the same identical coefficient 0.457 as the mean of the two given for the 3 feet and 6 feet overfalls. Castel's experiments for overfalls, with channels attached, sloping 1 in 13.3, give at the mean 0.527; the mean of those in § 73, sloping 1 in 12, is 0.507. In these also, if we take out the particular heads of 0.164 feet in the former, and 0.166 in the latter, we have the respective coefficients 0.530 and 0.532.

79. The overfalls having the bar, in the first set of experiments of plank, and in the second set at Chew Magna, of cast-iron, each 0.166 feet thick, and with square top edges, represent a very common structure for waste weirs, tumbling-bays, &c., on artificial canals and feeders. The average of the first set, with charges from 0.083 to 1.166, and 10 feet length of overfall, gives 0.556; that of the second set is as high as 0.723.

The plan and longitudinal section of the channel of approach is evidently more favourable in the latter case than the former; and the velocity of the approaching water must also have been considerable, from the circumstances mentioned in § 76. The very low coefficient 0.556 is not, however, readily to be accounted for; nor is it easy to assign a reason why, in the first set, the change from a thickness of 0.0052 feet to 0.166 feet in the overfall bar should lower the coefficients to such an extent, every other circumstance being apparently the same.

If we look to the coefficients of particular experiments, as, for instance, under charge 0.083 feet, length 10 feet, we find 0.808 with overfall 0.0052 feet thick, and only 0.435 with that whose edge was the plank 0.166 feet thick; and again, under charge of 0.75 feet, with 10 feet lengths, we have 0.529, for overfall 0.0052 thick, and 0.558 for that 0.166 feet thick,—that overfall having in this case the lesser coefficient, which had in the first instance one nearly double; again, with the

3 feet overfalls, head 0.50, the coefficient is the *same in both*, —namely, 0.592.

80. The following Table, compiled from various sources, exhibits at one view the results of different experimenters. Figs. 57 to 63 are diagrams illustrating the same.

Overfall, 0.5 feet long.—Smeaton and Brindley.												
Heads,			.083	.1042	.1146	.1354	.1927	.2604	.4166	.4687	.5417	
Coeffs.			.713	.681	.654	.638	.636	.602	.609	.571	.633	
Overfall, 1.533 feet long.—Du Buat.												
Heads,						.1482		.2666	.3887		.5627	
Coeffs.						.648		.624	.627		.630	
Overfall, 0.656 feet long.—D'Aubuisson and Castel.												
Heads,			.098	.131	.164	.196	.262	.328	.393	.459	.524	.590
Coeffs.			.632	.624	.620	.617	.616	.617	.620	.624	.628	.633
Overfall, 0.6458 feet long.—Poncelet and Lesbros.												
Heads,	.033	.066	.099	.1332	.1998	.2664	.333	.5	.666	.75		
Coeffs.	.636	.625	.618	.611	.601	.595	.592	.590	.585	.577		
First Set.—Overfalls, 3 feet and 10 feet long.—Simpson and Blackwell.												
Heads,			.083	.166	.25	.33	.416	.50	.583	.666	.75	
Coeffs.			.742	.738	.636	635	.625	.592		.580	.529	
Second Set.—Overfall 10 feet long.—Simpson and Blackwell.												
Heads,			.083	.166	.25	.33	.416	.50	.583	.666	.75	
Coeffs.			.608	.682	.725	.745	.780	.749	.772	.802	.781	
Overfall, 10 feet long.—J. B. Francis.—Lowell Experiments.												
Heads,				0.62	0.65	0.80	0.83	0.98	1.00	1.06	1.25	1.56
Coeffs.				0.622	0.622	0.623	0.625	0.625	0.622	0.627	0.623	0.62

The volume of water which passed into the measuring tank in the last experiments was between 11,000 and 12,000 cb. feet. In every case except the last three we may perceive that the coefficient decreases as the charge increases. Another exception may be found by referring back to the experiments of the first set,—overfall being a plank 0.166 feet thick,—in which the coefficients increase with the increase of the charges; the lengths being 3 feet, 6 feet, and 10 feet; and with the lengths of 6 feet and 10 feet attaining a maximum value at the charge of 0.583 feet, from which they slightly increase.

Experiments on the smaller differ from those on the greater "heads" in this, that they can be, and generally are, continued for a longer period of time: a measuring tank of a definite capacity being always part of an experimental apparatus, the smaller discharges may be much prolonged, and thus all errors, such as in the noting the commencement and end of an experiment, are diminished.

81. *Method of determining the coefficient from experiments.*—Smeaton's experiments were conducted by making observations upon the time in which a vessel of 20 cubic feet capacity was filled by the water flowing over a notch 0.5 feet long, and with the different charges given in the Table above.

Thus, with 0.1042 feet charge we should have, from the formula—

$$\frac{2}{3} lH\sqrt{2gH} = \frac{2}{3} \times 0.5 \times 0.1042 \times \sqrt{0.1042} \times 8.024 = 0.08995$$

cubic feet per second; but the experiment gives 20 cubic feet in 326 seconds, or (20 ÷ 326 =) 0.06135 cubic feet in one second, and—

$$0.08995 : 0.06135 :: 1 : 0.682 \left(= 1 \times \frac{0.06135}{0.08995}\right).$$

So also: Du Buat had a gauging reservoir to receive the discharged water, whose area was 108 square feet French; the water discharged by a notch (*reversoir*) 17 pouce 3 ligne long, with a head of 1 po. 8 lig., raised the surface of the reservoir above mentioned 1 po. in three minutes: hence the discharge per second was $15552 \times 5 \div 180 = 432$ cubic pouce, and the formula gives—

$$Q = \frac{2}{3} \times 17.25 \times 1.666 \sqrt{1.666} \times \sqrt{2g};$$

and as with this unit $2g$ is equal to 724, we have $\sqrt{2g}$ equal to 26.907, the resulting value is 66503 po. cubic, and 665.3 : 432 :: 1 : 0.648; and in all experiments in general, the cubic quantity discharged in the observed time is to be reduced to the quantity per second, by dividing the former by the time expressed in seconds; and this, the actual discharge, being divided by the result of the formula expressed in numbers, gives the coefficient by which the formula must be affected to make its results coincide with actual experimental results.

Mr. T. E. Blackwell used in his first set a gauging tank of a capacity of 444.39 cubic feet: we find with an overfall 3 ft. long, and a head of 0.083′ feet, that in 757 seconds the discharge is 137.91 cubic feet, i. e. $\frac{137.91}{757} = 0.182$ cubic feet per second, now

$$\frac{2}{3} \times 3 \times 0.083 \sqrt{0.083} \times 8.024 = 0.3836 \text{ cubic feet,}$$

and $0.3836 : 0.182 :: 1 : 0.4744 = m$. In the Table, § 73, these data give $m = 0.466$; but this is the average of three experiments, of which the above is one. With an overfall of 10 feet long, and a head of 1 foot, we have the discharge equal to 442.29 cubic feet in 15.5 seconds, i. e. $\frac{442.29}{15.5} = 28.535$ cubic feet per second, and $\frac{2}{3} \times 10 \times 1 \times \sqrt{1} \times 8.024 = 53.493$; therefore

$$\frac{28.535}{53.493} = .5334 = m.$$

82. Mr. Beardmore, in his "Hydraulic Tables," has used the formula—

$$(a) \quad . \quad . \quad . \quad . \quad . \quad . \quad Q = 214 \sqrt{H^3},$$

in constructing his Table for the "Discharge of Weirs or Overfalls." This formula is very nearly identical with that in § 64, second case; for as this (a) gives the discharge, not for any length, l, but for one foot in length only, and per minute instead of per second, as all the formulæ given in the present work, we must, to compare them, divide (a) by 60, and multiply by l: hence, as $214 \div 60 = 3.566$, we have—

$$Q = 3.566\, lH \sqrt{H} \left(= \frac{2}{3} mlH \sqrt{2gH}\right),$$

and consequently as—

$$\frac{2}{3} \times \sqrt{2g} \times m = 3.566, \text{ we have } m = \frac{3 \times 3.566}{2\sqrt{2g}} = 0.66654;$$

hence, we may write (a) thus,—

$$Q = \frac{2}{3} \times l \times 60 \times 3.566\, H \sqrt{H}$$

for any length l, and per second, and not per foot of length of overfall, per minute.

This author also remarks, "that the constant 214 is liable to some variation under unfavourable circumstances: for instance, when the weir is formed of a number of short bays, divided by vertical beams, grooved for sliding down the horizontal waste-boards to regulate the surface-level of top water. In these cases, the water passing the edges assumes the *vena contracta* form in each bay, and, consequently, the total width, L, of the opening should be reduced to obtain the true quantity of water passing. These, and other causes which may render the observations liable to error, must be treated with judgment, according to circumstances." . . . . "The best way of gauging for weirs is to have a post with a smooth head, level with the edge of the waste-board or sill: *to be driven firmly* in some part of the pond above the weir which has still water. A common rule can then be used for ascertaining the depth, or a gauge, showing at sight the depth of water passing over, may be nailed with its zero at the level of the sill of the weir. Among the conditions essential to a correct result are the absence of wind and current, a good thin-edged waste-board, the water having a free fall, and a weir not so long in proportion to the width above it as to wire-draw the stream; for in this case the water will arrive at the weir with an initial velocity due to a fall, which is not estimated in the gauging, and the result will in all probability be too small."

CHAPTER II.

FLOW OF WATER UNDER A VARIABLE HEAD.

83. *Flow of Water when the level is variable upon one or both faces of the orifice of discharge.*—When a reservoir, instead of being maintained constantly full, as we have supposed it to be hitherto, receives no supply, or receives less than it discharges through an orifice in the bottom, the surface of the fluid gradually descends, and the tank or reservoir is at length emptied. The laws of the discharge are in this case different from those which have been stated in the first chapter, and the questions to be resolved are of a different character.

The form of the vessels may be also, either prismatic—that is, of identical sections at every height of the surface—or having sides sloping at some known inclination.

84. *Ratio between the velocities at the orifice and in the vessel.*—Let us suppose that the fluid contained in a prismatic vessel be divided into extremely small horizontal sections, and that they descend parallel to each other, the particles of the fluid in each of the sections must then have the same velocity. This is the hypothesis of the parallelism of the horizontal sections, admitted, and perhaps too much extended, by many hydraulicians.

Let v be the velocity of the particles in the vessel; V that which they have at the orifice; A the horizontal section of the reservoir or vessel containing the water; S, or rather mS, that of the orifice; m being the coefficient of contraction, the volume of water which flows out in the indefinitely small time τ will be expressed by $mSV\tau$. During this same time the surface of the water will descend by a quantity $v\tau$, and the corresponding value of the volume of water will be $Av\tau = mSV\tau$, or $v : V :: mS : A$, giving an example of that hydraulic axiom mentioned § 16, p. 14,—namely, that the velocities are in the inverse ratio of of the sections.

85. *Head due to the velocity of the water at its point of discharge.*—The velocity V of the issuing fluid does not now maintain the same constant rate. It is uniform but for a given instant; for, besides being due to the actual head at the given instant, the velocity V is a consequence of the velocity v acquired during the descent of the parallel sections above men-

tioned: the two velocities acting in the same direction, from above downwards, their resultant is equal to their sum. Thus, if

$$H'\left(=\frac{V^2}{2g}\right)$$

be the generating height due to the velocity which the water has at its point of discharge, H always representing the actual head in the vessel, we shall have—

$$H'=H+\frac{v^2}{2g}=H+\frac{V^2}{2g}\cdot\frac{m^2S^2}{A^2}=H+H'\frac{m^2S^2}{A^2},$$

from whence we may deduce—

$$H'=\frac{H}{1-\frac{m^2S^2}{A^2}}=H\cdot\frac{A^2}{A^2-m^2S^2}.$$

When mS is small compared with A, as is generally the case, m^2S^2 will be inappreciable with regard to A^2, and may be neglected; in which case, $H'=H$, that is to say, the velocity of issue at any given instant is that due to the actual height of the water in the vessel at that same moment. In this chapter it is always assumed to be so, although the hypothesis of the parallelism of the horizontal sections, however admissible in their descent, does not hold good when they have arrived at the sphere of action of the orifice, the circumstances of the movement of the molecules of the fluid become then very complicated, and are indeed entirely unknown.

86. *Nature of the motion.*—Let M (Fig. 26) represent a vessel of water filled up to AB; let us divide the height from B to the orifice D into a great number of equal parts, B*a*, *ab*, *bc*, &c. Suppose, then, that a body, P, were impelled from below upwards with a velocity such that it rises to the point H, PH being equal to DB, and let us divide PH into the same number of equal parts.

In proportion as the body rises, its velocity will diminish, in such manner that when it shall have arrived successively at the points *a'*, *b'*, *c'*, the velocities will be respectively $\sqrt{Ha'}$, $\sqrt{Hb'}$, $\sqrt{Hc'}$ . . . 0, as is shown in works on the Elements of Mechanics. Recurring to the fluid contained in the vessel M, in proportion as it flows out, the surface AB is lowered; and when it shall have successively reached the points *a*, *b*, *c*, the respective velocities of the issuing water will be

(§ 85) as $\sqrt{\mathrm{D}a}$, $\sqrt{\mathrm{D}b}$, $\sqrt{\mathrm{D}c}$ . . . o, or, according to the construction, as their equals, $\sqrt{\mathrm{H}a'}$, $\sqrt{\mathrm{H}b'}$, $\sqrt{\mathrm{H}c'}$ . . . o; so that, in proportion as the vessel is emptied, the velocity of the discharge will decrease down to zero, following the same law as the velocity of the body impelled from below upwards, each being an example of an uniformly retarded motion; consequently, the discharge also will be governed by the same law.

It will be the same, also, in the descent of the surface of the water in the vessel, which will be uniformly retarded, its velocity being in a constant ratio to that at the orifice,—namely, as the section of the orifice to the area of the surface of the water.

87. *Volume discharged.*—According to the laws of a uniformly retarded motion, when a body, starting with a certain velocity, loses it gradually until it is reduced to zero, it only describes one-half the space it would have traversed in the same time if it had moved uniformly with the velocity with which it commenced the motion. Now the volume of water which flows out from any vessel until it is all discharged may be regarded as a prism, whose base is the orifice, and height the space which the first issuing particles would describe, with a uniformly retarded motion identical with that by which the discharge takes place; but if the same particles had always preserved their initial velocity (which is that due to the primary charge), the space described in the same time, or the height of the prism, and, consequently, the volume of water discharged, would have been doubled. Hence this theorem;—*The volume of water which passes through an orifice at the bottom of a prismatic vessel, receiving no supply, and therefore becoming empty, is only one-half of that which would be given during the time of complete discharge, if the flow had taken place under a constant charge equal to the primary.*

88. *Time which is required to empty a vessel.*—Let H be this charge; A the horizontal section of the vessel; T the time which it may require to be completely discharged. The volume of water discharged during this time—that is to say, all the water the vessel contains (above the orifice)—is $A \times H$. The volume, according to the theorem above, which would have been discharged in that time under the constant charge H, would have been $2\,(A \times H)$. This same volume, or the discharge during the time T, is also equal to $mST\sqrt{2gH}$.

We may use the Italic capital H instead of the Roman H, conventionally applied hitherto in this formula (§ 46), since the orifice is now supposed to be in the bottom of the vessel, and therefore H and H are identical. Equating these two values, we have—

$$2AH = mST\sqrt{2gH},$$

and solving for T, we have—

$$T = 2 \cdot \frac{AH}{mS\sqrt{2gH}},$$

and dividing above and below by $\sqrt{H}$, we have, finally—

$$T = 2 \times \frac{A\sqrt{H}}{mS\sqrt{2g}}.$$

If we represent by T' the time which the volume AH would take to flow out under the constant head H, we should have had (§ 14)—

$$AH = mST'\sqrt{2gH}, \text{ or } T' = \frac{A\sqrt{H}}{mS\sqrt{2g}},$$

consequently, $T = 2T'$; that is to say, *the time which a prismatic vessel takes to be completely discharged is double that in which the same volume would flow out, if the head had remained constantly the same as it was at the commencement of the discharge.*

89. *Time which the surface of the water takes to descend a given depth.*—Let t be the time sought in which the level descends the given vertical depth a: now the time in which the whole volume would be discharged is (§ 88)—

$$\frac{2A\sqrt{H}}{mS\sqrt{2g}};$$

the head at the commencement being H; and putting—

$$H - a = h$$

for the head at the end of t, we have the time in which the volume hA would be entirely discharged equal to—

$$\frac{2A\sqrt{h}}{mS\sqrt{2g}}.$$

Now, the time t, that in which the surface descends a height equal to a, is evidently the difference between the two expressions given above, that is—

$$(a) \; \ldots \; t = \frac{2A\sqrt{H}}{mS\sqrt{2g}} - \frac{2A\sqrt{h}}{mS\sqrt{2g}} = \frac{2A}{mS\sqrt{2g}}\left(\sqrt{H} - \sqrt{h}\right).$$

This proposition and that in § 88 give a method of determining by experiment the coefficient of contraction at an orifice, and have been used by Dr. Young for that purpose (Transactions, Royal Irish Academy, vol. ii., p. 81, &c.). *Vide* EXAMPLES to Chap. II., xxxiii. and xxxiv.

90. *Volume discharged in a given time.*—The above expression for the time which the water requires to descend any given height, by a simple transformation, gives both the value of a, and also the volume of water discharged during the given time: thus we have from (a)—

$$\frac{tmS\sqrt{2g}}{2A} = \sqrt{H} - \sqrt{h},$$

and—

$$\frac{tmS\sqrt{2g}}{2A} + \sqrt{h} = \sqrt{H};$$

hence,

$$\sqrt{h} = \sqrt{H} - \frac{tmS\sqrt{2g}}{2A},$$

squaring both sides of the former equation—

$$\left(\frac{tmS\sqrt{2g}}{2A}\right)^2 + 2\,\frac{tmS\sqrt{2g}}{2A}\,.\sqrt{h} + h = H;$$

hence—

$$\frac{tmS\sqrt{2g}}{A} \times \left(\frac{tmS\sqrt{2g}}{4A} + \sqrt{h}\right) = H - h.$$

Hence, substituting for $\sqrt{h}$ its value given above, and multiplying both sides by A, we have the discharge Q' for the given time—

$$Q' = (H - h)\,A = tmS\sqrt{2g}\left(\sqrt{H} - \frac{tmS\sqrt{2g}}{4A}\right).$$

91. *Mean hydraulic charge.*—A prismatic vessel (Fig. 27), receiving no supply, discharges water through an orifice, S, during the time T seconds, having at the commencement the head H, and at the end that represented by h; required the mean hydraulic charge H', by which, *ceteris paribus*, the same quantity of water would have been discharged: we have (§ 14)—

$$(b) \; . \; . \; . \; . \; . \; Q' = mS\sqrt{2g}\,T\,\sqrt{H'} = (H-h)\,A\,;$$

also § 89 (*a*)—

$$T = \frac{2A}{mS\sqrt{2g}}\left(\sqrt{H} - \sqrt{h}\right);$$

substituting this value of T in (b), we have—

$$Q' = mS\sqrt{2g} \times \frac{2A}{mS\sqrt{2g}}\left(\sqrt{H} - \sqrt{h}\right) \times \sqrt{H'} = (H-h)\,A.$$

Clearing of fractions, and dividing, we have—

$$\sqrt{H'} = \frac{H-h}{2\left(\sqrt{H} - \sqrt{h}\right)}, \text{ or } H' = \left(\frac{H-h}{2\left(\sqrt{H} - \sqrt{h}\right)}\right)^2,$$

$$\text{or } H' = \left(\frac{\sqrt{H} - \sqrt{h}}{2}\right)^2.$$

Cor.—If $h = 0$, then $H' = \left(\frac{\sqrt{H}}{2}\right)^2 = \frac{H}{4}.$

92. *Case in which the basin receives a constant stream during the time it is discharging.*—Let us suppose that a prismatic basin receives a constant supply (less than that discharged), and that we have to determine, in these circumstances, the time that the surface will require to fall a given height. Continuing the same letters in the same significations, let us, in addition, call the volume of water arriving at the basin in one second q, and let x be the height which the water descends in the given time t, and dx the height which the surface descends in the indefinitely small time dt, Adx will then express the volume of water which flows out during dt, if the basin received no supply; but as it does receive q in one second, and consequently qdt in dt, the volume of water

actually discharged will be $Adx + qdt$. This same volume, from the formula for the discharge ef water through orifices (§ 14), is expressed by $mSdt\sqrt{2g\,(H-x)}$; we shall therefore have—

$$(a) \quad . \quad . \quad . \quad Adx + qdt = mSdt\sqrt{2g\,(H-x)}\,;$$

and if we make $H - x = h$, and therefore $-dx = dh$, we have

$$(b) \quad . \quad . \quad . \quad . \quad qdt - Adh = mSdt\sqrt{2g}\sqrt{h},$$

which gives—

$$(c) \quad . \quad . \quad . \quad . \quad . \quad . \quad dt = \frac{-Adh}{mS\sqrt{2g}\sqrt{h}-q}.$$

In order to integrate this equation, we may put—

(d) . . . $mS\sqrt{2g}\sqrt{h} - q = y$, and thus—

$$dt = \frac{-A}{m^2S^2g}\left(dy + q\frac{dy}{y}\right),$$

the integral of which is—

$$t = \frac{A}{m^2S^2g}\,(y + q \text{ hyp. log } y) + \text{C}.$$

Determining the value of C for the commencement of the motion, when $t = 0$ and $x = 0$, and H also being equal to h, we have, substituting for y its value above,—

C equal to—

$$\frac{2A}{(mS\sqrt{2g})^2}\,\overline{(mS\sqrt{2g}\sqrt{H} - q + q \text{ hyp log } mS\sqrt{2g}\sqrt{H} - q)}.$$

Hence t is equal to—

$$\frac{2A}{(mS\sqrt{2g})^2}\left\{mS\sqrt{2g}(\sqrt{H} - \sqrt{h}) + q \text{ hyp log}\frac{mS\sqrt{2gH} - q}{mS\sqrt{2gh} - q}\right\}.$$

It is evident from this expression that when $q = 0$, that is, when no supply is flowing in, it becomes identical with that in § 89.

If we had to determine the height which the level of the water would descend in a given time, the question would be reduced to this other—namely, to find the charge h at the end of this time, and subtract it from H, the head, at the commencement of the discharge. To obtain h we must substitute successive values of it; i. e. of $(H-x)$, in the equation given above, and thus tentatively determine that which satisfies the equation.

93. *Case when the water is discharged over a weir.*—In the case when the water issues from the basin by an overfall, supposing that it receive no supply, we shall have, from what has been laid down in § 46 and § 55,

$$Adx = \frac{2}{3}\, ml\,(H-x)\,dt\,\sqrt{2g}\,\sqrt{H-x},$$

whence, by a method analogous to that which has been used above, we have—

$$t = \frac{3A}{ml\sqrt{2g}}\left(\frac{1}{\sqrt{h}} - \frac{1}{\sqrt{H}}\right).$$

94. *Reservoirs not being prismatic.*—We have hitherto considered only the particular case of prismatic basins or reservoirs: the determination of the time of discharge for any other form is much more complicated, and is even impossible in most cases which present themselves. The fundamental equation is, however, always—

$$Adx = mSdt\,\sqrt{2g\,(H-x)},$$

from whence we have—

$$dt = \frac{Adx}{mS\sqrt{2g\,(H-x)}}.$$

But here A is variable, and we must, in order to integrate, express A in terms of x, which can only be effected when we know the law by which A decreases, and in the cases where the basin itself is a solid of revolution, whose generatrix is known. In every other case it will be necessary to proceed by approximations and by parts. To this end, we must divide the basin or reservoir into horizontal sections of small depth. Each of these may be taken as prismatic, and we can determine the time it takes to be discharged by the aid of the formulæ given above. The sum of these partial times will give the time that the surface of the water takes to descend a height equal to the sum of the heights of the prisms.

95. *Flow of water when it is discharged from one reservoir into another.*

1st. In the case when the orifice is covered with water on both faces or sides, the levels remaining constant, the quantity discharged is then the same as if it had taken place into the open air under a charge $H - h$, equal to the difference of the charges upon each face; thus we have, representing by Q the discharge per second,—

$$Q = mS\sqrt{2g(H-h)}.$$

2ndly. Let us suppose that the level remains constant in the upper basin, and that the lower of a given area receives it without any loss; required the time which must elapse before it has reached the level of the upper basin or a given height. This problem is the exact inverse of that in § 89, in which the discharge took place into the air, and the surface of the water above the orifice descended with a motion uniformly retarded. In the present case, the surface of the basin, urged from below upwards with a force measured by the difference of the level between the two basins,—which decreases in the same proportion as the charge decreased in the former case,—rises with a motion uniformly retarded, and it will require the same time to traverse the same space under these similar pressures.

Let H represent the pressure or charge AC (Fig. 28) at the commencement, and h the charge AD at the end of the time t, A the horizontal section of the vessel being filled, and S and m as before,—we shall have, for filling up to DE,

$$t = \frac{2A}{mS\sqrt{2g}}(\sqrt{H} - \sqrt{h}).$$

and for that of filling up to AF we shall have—

$$T = \frac{2A}{mS\sqrt{2g}}\sqrt{H}.$$

These latter formulæ are of some importance: they serve to determine the time in which canal locks, &c., may be filled, and to assign the area of sluice-way required to fill in a given time.

96. 3rdly. *The level of the water being variable in each vessel.*—We come now to the third case that can arise, namely, when

two reservoirs of different level communicate with each other, each being limited in area and receiving no supply, and thus one surface descends as the other rises. Such is the case of the two basins K and L (Fig. 29), communicating by a wide pipe EF, provided with a sluice-door or cock at G. Before the opening of this sluice-door the level of the water is at AB in the first reservoir, and CO in the second. At the end of a certain time after the opening of the communication it has descended to MN in the first, and has ascended to PQ in the second. It is required to determine the relation between these two heights at a given time, or, *vice versâ*, from the given difference in the respective heights, to determine the time corresponding to a given discharge.

Let t equal the given time, BE $= H$, CF $= h$, NE $= x$, PF $= y$, $A =$ horizontal section of the first vessel, and $B =$ that of the second, $s =$ section of the pipe of communication: in the coefficient m we must include the resistance of the water passing through this pipe. Whilst the water has risen in the second basin by the quantity dy, during the instant dt, it will have lowered in the other by dx; and remembering that x diminishes while y and t increase, we have $Adx = -Bdy$ and (§ 14),

$$(a) \quad . \quad . \quad . \quad . \quad Adx = -ms\sqrt{2g(x-y)}\,.\,dt,$$

from whence—

$$(b) \quad . \quad . \quad . \quad . \quad . \quad dt = -\frac{Adx}{ms\sqrt{2g(x-y)}}.$$

The first equation being integrated, remembering that when $x = H$, $y = h$: we have—

$$(c) \quad . \quad . \quad . \quad . \quad . \quad Ax + By = HA + Bh;$$

solving for y we have—

$$y = \frac{A(H-x)}{B} + h;$$

and substituting this value of y in the preceding equation (b), integrating, and observing that $H = x$ when $t = 0$, we have—

$$t = \frac{2A\sqrt{B}}{mS\sqrt{2g}(A+B)}\left\{\sqrt{B(H-h)} - \sqrt{(A+B)x - AH - Bh}\right\}$$

If it were required to find the time in which the two surfaces would be at the same level, we should have from (c)—

$$x = y = \frac{AH + Bh}{A + B};$$

and, this value of x being substituted in the above expression for t, will give—

$$(d) \quad . \quad . \quad . \quad . \quad . \quad t = \frac{2AB\sqrt{H-h}}{mS\sqrt{2g}(A+B)}.$$

From whence it is evident that for the same value of $(H-h)$ t is the same whether A be the horizontal section of the basin that lowers, and B that of the other, whose surface rises, or, *vice versâ*, B that which falls, and A that which rises.

EXAMPLES AND PRACTICAL APPLICATIONS.

CHAPTERS I. AND II.

97. *Questions solved by means of the formula* $mS\sqrt{2g}\text{H} = Q$, *the charge on the centre being represented by* H.—(1.) In order to obtain a comparative view of the effects resulting from the use of the different coefficients for the discharge through orifices, given in §§ 18 to 43, to which we first confine our attention, let us take a circular orifice of 0.25 ft. in diameter, the area S being therefore $0.25^2 \times 0.7854 = 0.04909$ sq. ft., and determine:—First, the discharge through it in some given time, as 40 minutes, with a constant charge of, suppose, 9 ft. above the centre of the orifice; and, secondly, with the same orifice and charge, seek the different intervals of time required to discharge a given volume of water, as 1000 cubic feet. As the charge is so great compared with the diameter in the above data, we may use the formula (§ 14)—

$$Q' = mST\sqrt{2g}\,\sqrt{\text{H}},$$

in which H is the charge on the centre. In the first case mentioned above we calculate the value of—

$$ST\sqrt{2g}\,\sqrt{\text{H}},$$

which becomes $0.04909 \times 40 \times 60 \times 8.024 \times 3 = 2836.067$ cb. ft., and multiply it by the several values of m, as is done below. For the second case we have—

$$\frac{Q'}{S\sqrt{2g}\,\sqrt{\text{H}}} \times \frac{1}{m} = T;$$

the value of the first factor of the left-hand side is—

$$\frac{1000}{0.04909 \times 8.024 \times 3} = \frac{1000}{1.1817} = 846.24 \text{ cb. ft.},$$

which must be divided also by the successive values of m to obtain T, the time required to discharge the given quantity.

	Value of Q in 40 min. $m \times 2836.067$.	Value of T, or 846.24 cb. ft. $\div\, m$.		
		min.	sec.	
(1) m (§ 28) = 0.50	1418 cb. ft.	28	12	internal tube.
(2) m (§ 18) = 0.62	1758	22	45	thin plate.
(3) m (§ 34) = 0.82	2325	17	12	cylindrical adjutage.
(4) m (§ 40) = 0.95	2694	14	51	conical converging adjutages.
(5) m (§ 18) = 1.00	2836	14	6	form of *vena contracta* and conl. converging.
(6) m (§ 43) = 1.46	4140	9	39	conical diverging adjutages.

(II.) Required the discharge in six minutes, through a rectangular sluice 3 ft. by 1 ft., the side 3 ft. long being horizontal, the depth to the sill from the surface being 7 ft., and m being equal to 0.62.

Here

$$0.62 \times 3 \text{ sq.ft.} \times 8.024 \sqrt{6.5} = Q,$$

and

$$\sqrt{6.5} = 2.5495 \text{ may be taken equal to } 2.55,$$

hence

$$Q = 38.06 \text{ cb. ft. per sec.},$$

and

$$Q' = 6 \times 60 \times 38.06 = 13701.6 \text{ cubic feet.}$$

(III.) A reservoir having at full water a depth of 40 feet over the centre of the discharging sluice, whose area is 2 feet horizontal by 1.5 ft. vertical when fully opened:—Required the discharge at that depth, and also when the water has sunk to the heads, 30 ft., 20 ft., and 10 ft., the value of m being taken at 0.62 in each case,—we have—

$$S = 1.5 \times 2 = 3 \text{ sq.ft., and } \sqrt{40}, \sqrt{30}, \sqrt{20}, \text{ and } \sqrt{10},$$

being respectively 6.324, 5.477, 4.472, 3.162. We must multiply these numbers successively by—

$$0.62 \times 8.024 \times 3 = 14.92464,$$

which is the same in each. Hence, for 40 ft. head the discharge is 94.384 per sec.; for 30 ft., 81.742 cb. feet.; for 20 ft., 66.743 cb. ft.; and for 10 ft., 47.192, or the half of that for 40 ft.; 3.162 being necessarily half 6.324, as they are the roots of numbers in the ratio of 1 to 4. This question points out the fact that leakages of sluices in lock-gates, &c., increase far less rapidly than the head, being, in fact, as the square roots of the charges. (*Vide* Smeaton's Reports, vol. i., pp. 196–9.)

(IV.) What is the discharge through a circular pipe 4 ft. diameter in the embankment of a reservoir, the head upon the centre being 90 ft., m being taken equal to 0.60? In this case—

$$S = (4)^2 \times 0.7854 = 12.5664 \text{ and } \sqrt{90} = 9.487,$$

hence

$$0.6 \times 12.5664 \times 8.024 \times 9.487 = 573.9 \text{ cb. ft. per sec.}$$

(V.) A rectangular sluice, sides 4 ft. horizontal and 3 ft. vertical, having a charge of 20 ft. on the centre, is raised 1.5 ft.: required the discharge per sec., and also when fully opened. We have the value of S in the first instance one-half that in the second, but the heads are 20.75 ft. and 20 ft. respectively; and taking $m = 0.62$, we have first—

$$0.62 \times 8.024 \times 6 \times \sqrt{20.75}\ (=4.5552) = 135.97 \text{ cb. ft.};$$

and secondly,

$$0.62 \times 8.024 \times 12 \times \sqrt{20}\ (=4.472) = 266.97 \text{ cb. ft.}$$

The double of the former would be 271.94 cb. ft.

(VI.) In cities in which water is supplied at high pressure, and constant service, it is sometimes usual to give the water to manufactories and works through a very small orifice, perforated in a disk, which is closed up secure from any possibility of unfair interference. Calculate the discharge through an orifice 0.0089 in. diameter for 24 hours, the head being 129 ft. and m equal to 0.62; we have—

$$\log. m + \log. S + \tfrac{1}{2}\log.(2g) + \tfrac{1}{2}\log. \text{H} + \log. 86.400'' = \log. Q',$$

the log. of S being 2 log. 0.0089 + log. 0.7854,—we have thus—

$$Q' = 303.655 \text{ cb. ft.}$$

(VII.) Suppose the pressure on the mains to be 150 ft. of water, and the diameter of the orifice 0.02 ft.: required the quantity delivered in 24 hours, the coefficient of discharge being 0.62. The $\sqrt{150}$ being equal to 12.247, and—

$$S = (0.02)^2 \times 0.7854 = 0.00031416,$$

we have—

$$Q = 24^{\text{h}} \times 3600'' \times 0.62 \times 0.00031416 \times 8.024 \times 12.247$$
$$= 1653.7 \text{ cb. ft.}$$

(VIII.) What must be the diameter of the orifice to give 600 cb. ft. per diem, the head on the main being 100 feet?

Here

$$S = \frac{600\ (=Q)}{24 \times 3600 \times 0.62 \times 8.024 \times 10} = \frac{600}{4298300} = 0.0001396 \text{sq.ft.},$$

and as $S = d^2 \times 0.7854$, we have—

$$d = \sqrt{\frac{0.0001396}{0.7854}} \quad \sqrt{0.0000177744} = 0.004216 \text{ ft.}$$

As the exact adjustment of this diameter would be nearly impossible, the process is somewhat tentative.

(IX.) In the sluices constructed in tidal harbours for scouring away at low water the silt that generally accumulates in them, we obtain examples on a very large scale of the discharge of water through orifices.

This simple remedy for a defect that had rendered nearly useless some of the most important tidal harbours on the coast of England, which had not the advantage of any sufficient natural streams to keep them open, was introduced by the great Smeaton from his personal observation of the practice in the Low Countries (*vide* Reports, vol. ii. p. 202–209). A bank thrown across some part, covered at high tide, impounded the water allowed to enter during the rise of the tide, and which at low water is discharged very rapidly through sluices constructed in this embankment, the sills of which are at low water of springs, or as low as possible.

The practice subsequently fell into disrepute, as it was found that the area of the back-water was itself soon silted up; but the same engineer adopted the simple and efficient remedy of dividing the back water into two separate areas by a second bank at or about perpendicular to the first mentioned, and by occasionally using one of these to cleanse the other, they were both, as well as the harbour itself, kept clear. Ramsgate and Dover are well-known examples (*vide* Smeaton's Reports and Sir J. Rennie on Harbours); from which last-mentioned work we take an example from the description of Hartlepool Harbour, on the coast of Durham.

Each sluice was 3 feet wide and 6.33 feet high, having a charge estimated at 10 ft. on the average. From the detailed plans of these works given by Sir J. Rennie, we may consider the coefficient 0.600 applicable; hence—

$$0.600 \times 3 \times 6.33 \times 8.024 \times \sqrt{10} = 289.14 \text{ cb. ft. per sec.}$$

is the discharge for each sluice; and as it is also stated that the total area of the scouring sluices was 366 sq. ft., of which 24 sq. ft. were given by four sluices, each 3 ft. by 2, in the lock-gates, which communicated with the back water or slake, we have 342 sq. ft. left for those through the embankment;

and each of these being $3 \times 6.33 = 19$ sq. ft., we have their number 18, i. e., $342 \div 19$; and the discharge for one being 289.14 cb. ft., the total discharge is $18 \times 289.14 = 5204.52$ cb. ft. per sec., or 312,271 cb. ft. per minute. Now the back-water containing 15,420,000 cb. ft., it could be discharged in about 50 minutes ($15420000 \div 312271$). It is essential that the back-water should be discharged rapidly before the rising tide diminishes the force of the artificial scouring action.

(x.) The widely different statements as to the efficiency of hydraulic engines as prime movers, some being asserted to give as high as 80, and others 60 per cent. of the power used, may in some cases, perhaps, be traced to a false estimate of the quantity of water discharged upon the prime mover used; for unless this be actually gauged, it must be calculated, and some coefficient used. In the case of undershot wheels with sloping sluices, as in Poncelet wheels, the bottom and sides being in continuation of the channel of supply, the coefficient is 0.74 when the sluice is inclined 1 base to 2 height, and 0.80 when 1 to 1 (Claudel, Aide Memoire, p. 78, § 100). If we had taken it 0.62, and with a six feet fall, the sluice being supposed 6 ft. wide and raised 1 ft., we have—

$$0.62 \times 6 \times 8.024 \times \sqrt{6} = 73 \text{ cb. ft.};$$

and had the modulus of the machine been calculated to be 80 per cent. from this discharge, it would, in reality, be but 67 per cent. found by the proportion $0.74 : 0.62 :: 80 : 67$.

(xi.) If the modulus of a water-wheel be estimated at 88 per cent., with a coefficient of discharge of 0.65, the wheel being 7 ft. broad, and the sluice, which slopes at 1 to 1, raised 0.75 ft., the head being 5.5 ft.: required the true modulus.

Here $0.65 \times 0.75 \times 7 \times 8.024 \times \sqrt{5.5} = Q = 64.2$ cb. ft.; hence (as $0.80 - 0.65 = 0.15$), $Q(1 + 0.15) : 64.2 :: 88 : 76.56$ per cent., the true discharge on the wheel being 73.8, i. e. 64.2×1.15.

(xii.) *Determination of the relative level in two vessels communicating by a submerged orifice.*—Let a vessel or cistern, A (Fig. 30) receive a constant supply of water, and discharge it into another vessel, B, through the submerged orifice *a*, which finally discharges into the air freely: the orifice at *b* is 1.0 feet horizontal by 0.2 feet vertical, and the charge H upon the centre 1.25, the coefficient being taken equal to 0.62; hence we find the discharge $0.62 \times 8.024 \times 0.2 \times 1.118 = 1.11238$ cubic feet per second, which must be equal to the constant supply received by A, and transmitted by it through the orifice *a* to B. Now this orifice is 0.8 feet by 0.1 foot, being a sluice 0.8 feet wide, and raised 0.1 feet,—capable, however, of being raised to 0.5 feet; and hence the charge upon it, which must be reckoned from the surface of B, is equal to $\left(\frac{1.11238}{0.62 \times 8.024 \times 0.08}\right)^2 = 7.806$ feet; and—as we should expect—the square roots of these charges are inversely proportional to the areas of the orifices; that is, 2.794 : 1.118 :: 0.2 : 0.08.

Hence, if we suppose the constant supply to be so increased as to raise the surface of the water in A one foot above its level in the last case, we may determine the corresponding rise in B, and also the additional quantity that has been supplied. The total vertical height above the centre of *b* is now $1.25 + 7.806 + 1 = 10.056$, which number has to be divided into parts whose square roots have the ratio 0.2 to 0.08, that is, of the areas of the orifices. Now $(0.2)^2 : (0.08)^2$ being as 4.00 to 0.64, we have—

$$4.64 : 0.64 :: 10.056 : \frac{10.056 \times 0.64}{4.64} = 1.387$$

for the one part, i. e., the surface of B above *b*, and $10.056 - 1.387 = 8.669$ for the other, that is, of the surface of A above that of B, and the quantity received in A is now 1.172 cubic feet per second. The actual rise of 1 foot in A corresponded, therefore, to one of $(1.387 - 1.25 =)$ 0.137 feet in B. If we had supposed the surface of A *lowered* 1 foot, then we find that B descends 0.1388 ft., and the constant supply is now 1.05 cubic feet per second. Hence the total range of B is only 0.277 feet for the corresponding change of 2 feet in A; a result which points out the value of the second or lower regulating chamber in insuring a nearly constant head on the other orifice *b*, and consequently a discharge also nearly constant, though the really effective head in A may vary considerably: a very important application is described in Examples XXIII., XXIV.

(XIII.) The time of filling a lock on a navigable canal (Fig. 31) consists of two distinct intervals: one, the time of filling up to the centre of the sluices; the second, that of raising the surface up to the level of the upper reach. The length of a lock being 115.1 ft., and breadth 30.44 ft., the horizontal area is 3503.6 square feet, and the vertical depth from centre of sluice to lower reach 1.0763 feet, the charge being 6.3945 feet; hence, the cubic content of the lower portion, that is, the value of Q', is 3771 cubic feet; the area of the two sluices 2×6.766 sq. feet $= 13.532$ sq. feet; and the charge on centre, as above, 6.3945 feet; and the value of m, assumed by D'Aubuisson, is 0.548 ft. From some experiments on the Canal of Languedoc, it was found that when two sluices were opened in the gates, the discharge was not double that given when only one was used: it was found, in fact, to be about an eighth part less, which reduces m from 0.625 (§ 24) to 0.548. We have therefore—

$$\frac{Q'}{m\,.\,S\,.\sqrt{2g}\cdot\sqrt{\mathrm{H}}}=T=\frac{3771\text{ cb. ft.}}{0.548\times 13.532\times 8.024\times\sqrt{6.3945}}=25''.$$

98. *Determination of the charge necessary to give a certain quantity with a given value of S.*—To determine the head necessary to give a certain discharge, we have but to solve $Q=mS\sqrt{2g}\sqrt{\mathrm{H}}$ for H; and hence—

$$\left(\frac{Q}{mS\sqrt{2g}}\right)^2=\mathrm{H}.$$

(XIV.) Required the head necessary to give 7.85283 cb. ft. per sec. through an orifice 0.5 feet square, m being equal to 0.625. Here—

$$\left(\frac{7.85283}{1.24175}=6.324\right)^2=40\text{ feet},$$

or 2 (log. 7.85283 - log. 1.24175) = log. H; that is,—

$$2\,(0.8950245-0.09401 67)=1.6020156=\text{log. of } 40.$$

If the orifice had been 0.75 feet square, determine the charge necessary to give the same discharge as in the last example, namely, 7.85283 cubic feet per second. Here—

$$\left(\frac{7.85283}{2.821}=2.7837\right)^2,$$

and

$$2\log.\ 2.7837=2\,(0.4446224)=0.8892448,$$

giving

$$\mathrm{H}=7.748984\text{ feet}.$$

What additional head would each orifice require to discharge 10 cubic feet per second, the coefficient remaining 0.625?

Here $7.85283 : 10 :: \sqrt{40} : \frac{63.24}{7.85283}$ = log. 63.24 - log. 7.8528 (= 1.8009919 - 0.8950245) = 0.9059674 = log. of 8.053, which is the square root of the charge required, whose value is therefore 64.85 ft., and, deducting 40, we have the increase of head equal to 24.85 feet.

And, $7.85283 : 10 :: \sqrt{7.748984} : \frac{27.837}{7.85283}$ = log. 27.837 − log. 7.85283 (= 1.4446224 − 0.8950245) = 0.5495979 = log. of 3.5448, which is the square root of the charge sought, 2 × 0.5495979 = 1.0991958 = log. of 12.575, from which deducting 7.748984, we have the additional head in this equal to 4.826 feet.

(xv.) Calculate the head that is equivalent to the difference between the coefficients 0.600 and 0.950; that is, having the discharge under certain data, with $m = 0.950$; determine what additional head would be required to give the same discharge when $m = 0.600$. Thus, let the charge on the centre be 8.55 feet, the orifice circular and 0.045 feet diameter, and so nearly the form of the *vena contracta* that the coefficient rises to 0.950; we have therefore—

$$S = (0.045)^2 \times 0.7854 = 0.00159, \text{ also } \sqrt{8.55} = 2.924,$$

and $Q = 0.95 \times 0.00159 \times 8.024 \times 2.924 = 0.03544$ cb. ft., and the head necessary to give this discharge with $m = 0.6$ is found (as $m S \sqrt{2g} = 0.6 \times 0.00159 \times 8.024 = 0.007655$) by—

$$\left(\frac{Q}{m \,.\, S\sqrt{2g}}\right)^2 = \left(\frac{Q}{0.007655}\right)^2 = \text{H, or, by logarithms,}$$

$$\left(\frac{0.03544}{0.007655}\right)^2 = (\bar{2}.5494937 - \bar{3}.8839452) \times 2 = 1.331097$$

$$= \text{log. } 21.43 \text{ feet.}$$

Thus 21.43 − 8.55 = 12.88 feet is the additional head or pressure required to discharge the same volume of water through the orifice in a thin plate that was discharged with 8.55 feet pressure through an orifice nearly of the true form. Thus, the accelerating force due to this form, when compared with the thin plate, is measured by a pressure equal to more than one-third of the weight of the atmosphere.

99. ***Results of the suppression of the contraction on part of the perimeter***, §§ 25 *to* 26.—A sluice 3 feet square, and with a charge on the centre of 12 feet, has, from the thickness of the frame, the contraction suppressed on all sides when fully open; but when partially opened, the contraction exists on the upper edge, that is, against the bottom of the gate, which is formed of a thin plate of metal. Required the discharge when opened 1 foot, 2 feet, and fully opened.

(XVI.) When opened 1 foot high, the total perimeter is 8 ft., and the part on which the contraction is suppressed is 5 feet: hence—

$$\frac{n}{p} = \frac{5}{8}.$$

Hence, from the formula (§ 26)—

$$m \,.\, S \,.\, \sqrt{2g}\,\sqrt{\mathrm{H}}\left(1 + 0.152\frac{n}{p}\right),$$

m being 0.608, we have for the discharge—

$$0.608 \times 3 \times 8.024 \times \sqrt{13}\left(1 \times 0.152\frac{5}{8}\right) = 57.77 \text{ cb. ft.}$$

per second; the two last factors being 3.605 and 1.095.

When opened 2 feet high, the total perimeter is 10 feet, and the contraction suppressed on 7 feet; so that—

$$\frac{n}{p} = \frac{7}{10}.$$

and—

$$0.152 \times \frac{7}{10} = 0.106,$$

also $\sqrt{\mathrm{H}} = \sqrt{12.5} = 3.5355$. Hence the discharge in this case is $0.608 \times 6 \times 8.024 \times 3.5355 \times 1.106 = 114.45$ cubic feet per second.

When the sluice is fully opened, the total perimeter is 12 feet and $\frac{n}{p} = 1$; so that the discharge is—

$$0.068 \times 8.024 \times 9 \times 3.464 \times 1.152 = 175.9 \text{ cb. ft. per sec.}$$

100. *Questions upon* §§ 44 *to* 52, *formulæ* $\frac{2}{3}ml\,H\sqrt{H}\sqrt{2g}$ *and* $\frac{2}{3}ml\sqrt{2g}\,(H\sqrt{H} - h\sqrt{h}) = Q$.—We derive from Captain Baird Smith's work on "Italian Irrigation" many examples whose solution is given by the formulæ in these sections. It appears in the irrigated districts to be a matter of great importance to determine both a unit of volume of water, and also some means of measuring it so as to regulate the due supply to each proprietor, that, on the one hand, the Government, or individuals having the ownership or manage-

ment of the canals of irrigation, may not be defrauded; nor, on the other, the proprietors of the land to be irrigated suffer any injustice. Many very different units are found in use at present. In the earlier periods of the construction of these canals, a fixed *area* of orifice, opened in the side of the canal, was alone used, without any reference to the head or charge under which the water issued: to this was subsequently added the condition of a fixed charge as well as a fixed area, but without any mechanical arrangements for insuring this constant pressure, which, from the continual variations in the level of the surface of the water in the canal, was absolutely requisite; however, unless the excess or deficiency complained of was more than the eighth part of the total volume specified in the grant, no complaint was admitted. A very simple and beautiful contrivance, founded on the effect of the double cistern on the discharge (§ 97 (XII.)), and described below, is now, however, in use on many of these works of irrigation, which meets this difficulty, sufficiently, at least, for practical purposes.

101. Taking these different measures or units in the order of date, as nearly as possible, it appears that on the Canal of Caluso the unit called *ruote*, or wheel, was defined to be the quantity passing through an opening whose area is 1 foot square,—this foot being equal to 1.6702 feet lineal English,—the upper edge of the outlet being what is locally termed a *fior di acqua*, or level with the surface of the canal or reservoir, the discharge hence taking place under no pressure. The volume discharged by the *ruota* is estimated by the Piedmontese engineers at 12.05 cubic feet English per second; but obvious sources of error and discrepancy arise from the reference being solely to the superficial area: first, on theoretical considerations; and secondly, on practical. For we may evidently have a number of orifices with very different discharges and figures, all fulfilling the condition of a constant area. The Piedmontese foot being divided into 12 inches, the value of an inch is 1.6702 English inches. The following short Table, limited to whole numbers of inches, will illustrate this point (all in Piedmontese inches):—

Linear dimensions applicable to a Ruote.

	Height *H*.		Breadth *l*.		Constant area.	Perimeters.	Ratio of sides.
1.	12	×	12	=	144	48 ...	1 : 1
2.	16	×	9	=	144	50 ...	1 : 0.5625
3.	18	×	8	=	144	52 ...	1 : 0.444′
4.	36	×	4	=	144	80 ...	1 : 0.111′
5.	48	×	3	=	144	102 ...	1 : 0.062
6.	72	×	2	=	144	148 ...	1 : 0.0277′

On examining this Table, we see the heights vary from 1 to 6, and the perimeters from 1 to 3, the area remaining constant. Many outlets are instanced, showing that this unit was actually used with a great variety of figure in many grants of water. But, secondly, a practical objection against any measure whose outlet is level with the supplying surface arises from the fact, that this surface is always changing, not only from the variations in the level of the river supplying the canal, but from the hourly changes in the demand for water from the canal by the various land-owners along its length.

(XVII.) Calculate the discharge which would be given by Nos. 1, 3, 5, and 6, of the above Table,—m being taken equal to 0.62, the foot being 1.6702 feet English, we have $\frac{2}{3} \times 0.62 \times 8.024 = 3.315$, and $l \times H = 2.789$ sq. ft. common to each; and thus—

For No. 1, we have $3.315 \times 2.789 \times \sqrt{1.6702} = 12.00$ cb. ft.
„ No. 3, „ $3.315 \times 2.789 \times \sqrt{2.505} = 14.63$ „
„ No. 5, „ $3.315 \times 2.789 \times \sqrt{6.6808} = 23.90$ „
„ No. 6, „ $3.315 \times 2.789 \times \sqrt{10.02} = 29.26$ „

Or generally, the discharge varying as $l \, . \, H \, . \, \sqrt{H}$, and $l \, . \, H$ being constant, it is evident that it increases as $\sqrt{H}$; so that, by increasing the depth indefinitely at the expense of the width l, we increase the discharge. Thus, let $H = 16$, the log. of $\frac{2}{3} mlH \sqrt{2g}$, that is, of $(3.315 \times 2.789 =) \, 9.2455$ being 0.9659304, we must add to it half the log. of H for the log. of the discharge: half the log. of 16 is 0.6020600, and adding, we have $1.5679904 =$ log. of 36.982 cb. ft.

102. *Questions on the formula* $\frac{2}{3} ml \sqrt{2g} \, . \, (H \sqrt{H} - h \sqrt{h}) = Q$, *the Italian dimensions being all reduced to English measures.*

(XVIII.) Ignazio Michellotti having determined to modify the mode of measuring a *ruote* introduced by his father, F. D. Michellotti, which had the upper edge level with the surface of the supplying canal, and was estimated to give a discharge equal to 11.83 cb. ft. per sec., defined the *uncia* or inch of water to be that flowing through an orifice 0.5567 feet high, 0.41755 ft. wide, and having a pressure on the upper edge of 0.5567 ft. This he supposed would give the twelfth part of 11.832 cb. ft., or 0.986. Calculate its true value: m being 0.600, we have then $H = 0.5567 + 0.5567 = 1.1134$ feet, and $l = 0.41755$ feet. *Ans.* 1.02 cb. ft.

(xix.) The measure used on the Canal Lodi was defined to be 1.12 ft. by 0.12416′ ft. wide, with a charge on the upper edge 0.32 ft., and these dimensions were supposed to give 0.77 cb. ft. per sec. Here $H = 1.12 + 0.32 = 1.44$ ft., and $l = 0.12416'$ ft. *Ans.* 0.6165 cb. ft. per sec.

(xx.) That used on the canal of Cremona was 1.31816′ ft. high by 0.131 ft. wide, having a head also 0.131 ft., and estimated to discharge 0.88 cb. ft. Hence—

$$\tfrac{2}{3} 0.6 \times 0.131 (1.449 \sqrt{1.449} - 0.131 \sqrt{0.131}) 8.024 = 0.715 \text{ cb. ft.}$$

(xxi.) That of Crema was 1.276 ft. high, 0.1275 ft. wide, a charge of 0.255 ft.: calculate the discharge.
Ans. 0.7225 cb. ft. per sec.

(xxii.) The Sardinian Civil Code determines the unit in which all grants of water should be expressed thus:—"The measure or *modulo* (Fig. 32) is that quantity of water which, under simple pressure, and with a free fall, issues from a rectangular quadrilateral opening, so placed that two of its sides shall be vertical, having a breadth of 0.6562 ft. (English measure), and a height also of 0.6562 ft. It shall be opened in a thin wall (or plate-*parete*), against which the water stands, with its upper surface perfectly free, at a constant height of 1.3124 ft. (= 2 × 0.6562) above the lower edge of the outlet." It is required to calculate the value of this unit in cubic feet per second. We have therefore $l = 0.6562$, and H and h, being 1.3124 and 0.6562 respectively—

$$\tfrac{2}{3} \times 0.6 \times 0.6562 (1.3124 \sqrt{1.3124} - 0.6562 \sqrt{0.6562}) 8.024 = 2.046 \text{ cb. ft. per sec.}$$

When grants are made for more than one *module*, the only dimension which varies is the breadth of the outlet, the height and pressure remaining in all cases invariable; two *modules* would have a breadth of outlet of 1.3124 ft., three would have 1.9686 ft., and so on.

103. ***Description of a Piedmontese outlet*** ("Italian Irrigation," pp. 21, 22, vol. ii.).—"AB (Fig. 33) is the transverse section of the supplying canal; the first part of the measuring apparatus is the sluice, which consists of masonry, side walls, and a gate of wood, working vertically. The dimensions of this primary outlet are not rigidly fixed, its object being merely to admit a larger or smaller supply into the chamber CD. The sluice is established in the bank of the canal, at such point as

may be fixed upon by the canal authorities, or most convenient for the land-owner. Its sill is sometimes on the same level as the canal bed, sometimes above it, and very frequently as represented in the diagram. There is a fall in front of the outlet, so as to draw the water towards it. For a length of from 40 to 50 feet from the sluice, the bed of the channel is made perfectly horizontal, paved with masonry or cut stone, the upper surface of which is on the same level as the sill of the sluice. At a distance from the outlet, ranging from 16 to 32 feet, is fixed the partition or slab of stone *c d*, in which the regulating or measuring outlet *e f* is cut, the height of which is fixed at 0.56 ft., while the breadth varies with the number of units or inches to be given, each inch being represented by 0.42 ft. of breadth. The lower edge of the measuring outlet is ordinarily placed at 0.819 feet above the level of the flooring of the chamber CD. A small return cut in the inner face of the slab, at a height of 0.28 ft. above the upper edge of the outlet, indicates the constant level of the water necessary to insure the established pressure. This height is maintained by the raising or lowering, as may be requisite, of the sluice at the entrance of the chamber.

(XXIII.) Calculate the value of a grant of three inches of water from this structure. We have $H = 0.56 + 0.28 = 0.84$; hence—

$$3 \times \tfrac{2}{3} \times 0.6 \times 0.42 \times (0.84\sqrt{0.84} - 0.28\sqrt{0.28})\,8.024$$
$$= 2.514 \text{ cb. ft. per sec.}$$

104. *Description of the modulo magistrale of Milan.*—This module, as applied upon the Naviglio Grande, which, in a course of 31 miles from its head on the River Ticino to the city of Milan, distributes 1851 cb. ft. per second, is in its principle identical with that already described (§ 103). For the interesting history of this canal, and the gradual improvements in the management of the grants of water, we refer to "Italian Irrigation," vol. i. pp. 203, 228; vol. ii. pp. 36, 56. The honour of the discovery is due to Soldati, of Milan, about the year 1571, who invented it in answer to an invitation from the magistracy of that city to architects and engineers to design a measuring apparatus.

The unit fixed upon, called the *oncia magistrale*, had the following dimensions (Fig. 34) :—Height, 0.655 ft.; breadth, 0.3426′ ft.; with a constant pressure of 0.32944 ft. above the upper edge of the outlet. When one outlet is designed for the discharge of several water-inches, the breadth only varies, in the proportion of 0.3426′ ft. for each additional water-inch, the height and pressure remaining constant, as in Fig. 35, which shows an outlet for six water-inches. The outlet is cut with care in a single slab of stone. To preserve it from being tampered with, an iron rim is fixed upon it, of the exact dimensions corresponding to the discharge. They ought invariably to be cut in a simple plate, with no arrangement of any kind to increase the volume beyond that due to pressure alone. The thickness of the slab varies somewhat with the dimensions of the outlet; but in a rigidly exact *module* this dimension should be fixed as well as all the others. These are the conditions applicable to the measuring outlets, the discharge from which is—

$$\tfrac{2}{3} \times 0.6 \times 0.34266'(0.98444\sqrt{0.98444} - 0.32944\sqrt{0.32944})\,8.024 = 0.866 \text{ cb. ft.}$$

To illustrate the other arrangements of the *modulo*, the plan and section (Figs. 36 and 37) are given from the same work.

The sluice AB (Fig. 36) is placed on the bank of the canal of supply, with the sill CD (Fig. 37) on the same level as the bottom of this canal. It is formed of two side-walls or cheeks, of good masonry, in brick or stone, with a flooring generally of the latter material. To prevent erosive action, the bed of the canal, for such distance as the force of the current may render necessary, is paved with slabs of stone or boulders, both above and below the head. The sluice-gate is usually made of the same breadth as that of the measuring orifice GH (Fig. 37), while its height is regulated by that of the sluice itself. The sluice-gate or *paratoja* IK (Fig. 37) works in grooves, and is fitted with a rack and lever, by which it can be readily raised or depressed at pleasure. As the surface level of the canals of the Milanese varies comparatively little, the upright of the sluice has a small catch in iron or wood attached to it, by which it is kept at a fixed height, corresponding to the requisite pressure on the original orifice GH (Fig. 37). This little catch is locally termed the *gatello;* and as it is provided with a lock and key, the latter of which is intrusted to the guardian of the canal, the proprietor of the water-course supplied through the *module* is supposed to be restricted to his legitimate supply, and probably is so within reasonable limits, provided always that the guardian is incorruptible. In the rear of the sluice-gate, at the head, is placed the first chamber LM (Figs. 36 and 37), called the *tromba coperta*, or covered chamber. Its length is equal to very nearly 20 feet, with a breadth varying according to the size of the head-sluice, which it exceeds by the fixed quantity of 0.82 ft. on each side, or 1.64 on the entire breadth. The bottom of the covered chamber DH (Fig. 37) is formed with a slope to the rere, the height H*h* being 1.3125 ft. English: its object is to diminish the velocity with which the water reaches the measuring outlet GH. Further to assist in effecting this object, the perfect *modulo* is provided with a horizontal top of stone slabs or planks, called the *cielo morte*, the under surface of which is at precisely the same height as the water ought to have over the outlet GH, so as to secure the fixed discharge, that is, 0.32944 ft. above the upper edge of GH. It is found that this does reduce the irregular motion of the water, and so tends to secure the great object of the *modulo*, that the discharge take place under simple pressure, and without antecedent velocity. To admit of ready inspection of the height of the water within the covered chamber, the following arrangements are made:—The entrance to the chamber is covered with a stone slab of convenient thickness, shown in section at E (Fig. 37), the lower surface of which is precisely on the same level as the upper edge of the outlet GH. The height of the slope H*h* being 1.3125 ft., and that of the outlet GH being 0.655, the surface of the slab at E should be 1.9675′ ft. above the sill of the head CD. An open groove LD is made in the masonry, large enough to admit a graduated rod or measure; and when the water stands at a height of—

$$(1.9675' + 0.3234 =)\ 2.297 \text{ ft.}$$

above the sill at D, it is known that the proper head of pressure exists at GH. As it is found to be greater or less, the sluice is depressed or raised, so as to adjust the pressure to the fixed standard. The slab of stone in which the measuring outlet is cut being fixed at GH (Figs. 36 and 37), immediately in rere of it there is placed the *tromba scoperta*, or open chamber. Its breadth at N (Fig. 36) is two local inches (0.3275 ft. English), greater on each side that that of the measuring outlet, or on both sides 0.6550 ft. Its total length NO is very nearly 17.75 ft. English. Its side-walls, which are perpendicular, like those of the covered chamber, have a splay outwards, so that the breadth at O is 0.9825 ft. greater than at N, or 1.31 ft. in excess of that of the regulating outlet GH, being the same as that of the covered chamber throughout. To insure the free run of the water from GH, the flooring of the open chamber has a drop or fall of 0.1633 ft. at H, and an equal quantity distributed uniformly between H and O (Fig. 37). There is therefore a total fall from the under edge of the measuring outlet to the end of the open chamber of 0.3275 ft., or, as the length is 17.72 ft., of 1 in 54. When the water reaches O, it enters the channel of distribution for the use of the consumers: generally the point O, and the bed of the channel, which is carried on at the usual inclination, are upon the same level, but sometimes there is a fall.

105. From the preceding details, it appears that the *modulo magistrale* has a total length of nearly 37.75 ft. English, and a breadth variable according to the quantity of water it is intended to measure. If a single water-inch, for instance, be granted, the breadth of the covered chamber would be 2.12835 feet, and that of the open chamber 1.145835 feet at its upper, and 2.12835 at its lower extremity. The flooring of the former rises 1.31 ft. to the rere, while that of the latter falls 0.3275 ft. in the same direction. It is essential to the effective operation of the regulating sluice in the *modulo magistrale* that there should be a difference of level between the water in the canal and in the apparatus of at least 0.655 ft.; and as the height of water in the latter must be 2.297 ft., the depth of water in the canal of supply must necessarily be not less than the sum of these numbers, or 2.952 ft., very nearly 3 ft.

It is curious to reflect that this apparatus was invented empirically by Soldati, in 1571, so many years before the discovery of the Toricellian theorem, which must be placed in the year 1643, when that philosopher showed that the velocity of running water was identical with that of falling bodies, the foundation of all our knowledge of Hydraulics. This is not the only instance in which the practical sagacity of the engineer has anticipated the discoveries of theory.

Two essential objects are fulfilled by the arrangements just detailed:—1st. To maintain on the measuring outlet a constant pressure; and 2nd. To make this pressure as much as possible the sole force influencing the discharge, that is, that the water have no velocity antecedently. The first is secured by the mechanical arrangements at the head,—the sluice with its rack, lever, &c., and to a certain extent the *cielo morte*. By raising or lowering this sluice the level of the water in the covered chamber is maintained, independent of the variations in the surface of the external canal. The second by the interior arrangements,—the covered chamber with its fixed top, and floor sloping up to the outlet; while the free passage of the water is secured by the open chamber, with its small fall at the head and continued inclination at the bottom.

106. The differences in the estimates of the quantity of water discharged by the *modulo magistrale*, as given by different Italian engineers, are very remarkable, considering the great attention that has been paid to the theory and practice of Hydraulics in that country. De Regi gives it as 1.42 cb. ft. per sec.; Breschetti states the average result of experiments on the Muzza Canal to give 1.57 cb. ft. per sec.; Mazzeri estimates it as low as 1.21 cb. ft.; Brunacci at 1.46; while the Department of Public Works in Lombardy considers it equal to 1.64 cb. ft. per sec. The extremes, we see, are 1.21 and 1.64 cb. ft. per sec., —a difference of 0.43 cb. ft., between a third and fourth of the total discharge. Captain Smith accounts for this great difference by stating—" That the estimate of the Government is founded on the experience of the results on the great canals, where the outlets are almost uniformly of large dimensions" (pp. 222, 223, vol. i.). Now it is certain that, all other circumstances being alike, the quantities of water discharged from large are proportionally greater than those discharged from small outlets. Hence the *oncia magistrale*, as determined by experiments with the former, has a decidedly higher value than when determined by the latter.

The cause of this is clear. To give a discharge of, say, six water-inches, the breadth of the outlet is made six times that for one inch, the height and the pressure remaining in both cases the same. The proportion between the sectional area and perimeter of the outlets becomes, however, materially altered, and the influence of the perimeter in effecting the contraction of the vein diminishes gradually as the size of the outlet increases; and in a similar proportion the discharge becomes greater. To elucidate this, it may be remarked, that in an outlet for one *oncia magistrale* the ratio of the section to the perimeter is as 1 to 23.33; for two, as 1 to 16.66; for four, as 1 to 13.33; for eight, as 1 to 11.66; for ten, as 1 to 11.33, or about half what it is for one *oncia;* for twenty *oncia*, as 1 to 10.66, and so on; and there are real differences of discharge due to the variable ratios now given.

Very serious pecuniary loss may consequently be the result to the proprietors of the canal or the consumers of the water. It appears (vol. i. pp. 226, 227) that for summer irrigation each cubic foot per second is capable of irrigating 61.8 acres, and that the annual rent of this quantity, summer and winter, is £13 5*s.*; the difference of 0.43 cb. ft. between the highest and lowest estimate of the discharge of the *modulo magistrale* is worth £5 13*s.*, and would irrigate 26 acres at the above rate.

The recognition of the differences between the discharges of large and small outlets was very early made in Lombardy. In the *module* of Cremona, invented in 1561, no single outlet was allowed to exceed 1.31 high by 3.18 broad, equal to about 12 or 13 water-inches. In the Milanese single outlets have been restricted for nearly three centuries and a half to discharges of from 9 to 12 *once*. In Piedmont they have been more careful, and have there limited single outlets to 6 *once*, which, by general consent, seems to be the most approved size for diminishing to the utmost the error due to the inequality of discharges from large and small openings. For practical purposes, therefore, and taking the mean of the various estimates of the value of the *oncia magistrale* just adverted to, it may be considered as equal to very nearly $1\frac{1}{2}$ cb. ft. per sec.

107. Another mode of insuring a constant discharge through an orifice having a charge subject to variation has been brought into use by the late Mr. Thom, an hydraulic engineer of great eminence. It attains this object by mechanical means chiefly. Figs. 38 and 39, taken from the description of the Gorbals Waterworks, near Glasgow (Practical Mechanics' Journal, August, 1848), will serve to explain this method. Fig. 40 represents a transverse section of the embankment of the reservoir. L, L is the level of the surface of the impounded water, and *l*, *l*, that of the receiving basin below, supplied from the reservoir by a pipe through the base of the embankment. Now should the quantity drawn off by the town or mill to be supplied increase, then the level of the surface *l*, *l* will descend; and the regulating apparatus must be of such construction that it may permit a larger quantity to pass through the pipe, and *vice versâ*.

Again, if the level of *l*, *l* should remain constant, and, from an increased or diminished supply, that of the reservoir L, L rise or fall, then this apparatus should be so constructed as to adjust the orifice of the discharging main pipe that it deliver only that constant quantity carried off from the receiving basin, and needed for the works.

Fig. 38 gives a longitudinal section of the detail of the regulator: *d* is a movable cast-iron cylinder or float attached at top to a chain passing over the pulley or wheel *e*, and surrounded by a fixed cylinder of a diameter slightly larger, containing water, and represented in section at *e*. The other end of this chain is fixed to the bent lever *b*, working freely on a stud carried by two cast-iron brackets screwed to the extremity of the pipe which passes through the base of the embankment of the reservoir, and terminates in a square mouth-piece, faced to receive a square hinged flap-valve, *a*, which is retained in any desired position by the lower and shorter arm of the bent lever, which works against the back of the valve by an anti-friction roller at *v*; the inner cylinder must be loaded with weights sufficient to keep the flap-valve closed when the outer cylinder is quite empty.

Now if we suppose the water in the outer cylinder *e* to stand at the level *s s*, the cast-iron float being immersed to a certain depth below this surface, part of its weight, acting by the chain upon the bent lever *b*, will press against the square flap-valve, and thus partly close the mouth of the main-pipe, restricting the discharge through it to the desired quantity. Suppose, then, that from any circumstances this discharge should become too small, and therefore the surface *l l descend*, it will then be necessary that the self-acting apparatus should be such as to permit the valve to open, and therefore, also, the cast-iron float to *rise*, which it will do if the water-level in the outer cylinder be made to rise; for then the cast-iron float becomes specifically lighter, and presses with a less force upon the valve *a*, which immediately yields to the pressure of the water issuing through the discharge-pipe, and thus permits a greater quantity to escape.

If, on the other hand, the quantity discharged had been too great, and thus the surface *l l rise*, it will be necessary that the cast-iron float *descend*, and thus press the flap-valve closer upon the square face of the discharge-pipe. This it will do if the water in the outer cylinder be made to fall; for thus the float becomes specifically heavier, and sinks, closing the flap-valve *a*: so that we have to devise such mechanical arrangements that when the discharge is too small, the water surface in the cylinder *e* shall rise, and when too great that it shall descend.

This is effected in the following manner:—A small closed cistern, *g*, is placed at the side of the portico of the entrance door of the building; this is supplied with water by a horizontal pipe, *r*, in communication with the vertical pipe, *h*, placed on the discharging main for the escape of air, which would otherwise collect within it, and greatly impede the discharge.

In all cases of discharge of water through pipes, care must be taken that the air which may collect be readily let off.—*Vide* Buck's Account of the Montgomeryshire Canal Lock; Simms on Public Works of England, p. 8.

The pipe, *h*, must necessarily be carried up the slope of the embankment, and communicate with the air above the level of the highest water in the reservoir. The cistern, *g*, is thus kept constantly supplied with water, and a communication is formed between it and the bottom of the cylinder, *e*, by the pipe, *k*. In the vertical part of this pipe are fixed two double-beat valves—described below—whose common spindle is fixed to the float *n*, placed in the receiving basin *l l*; now if the surface of the water upon which *n* rests should rise beyond the proper level, then this float, *n*, also rises, and, forcing up the spindle, closes up the upper or discharge valve from the cistern, *g*, and, as the valves are fixed on one spindle, of course simultaneously opens the lower one, so that the water which buoys up the float *d*, in the cylinder *e*, begins to flow out, and the consequent depression of the surface *s s*, causing *d* to descend, partially closes the flap-valve, *a*; and therefore the surface *l l* begins to descend, and with it the float *n*, which consequently opens the valve which had shut off the water from the cistern *g*, which, again receiving a supply, *d*, rises, and consequently the flap-valve opens, and thus very soon arrives at a nearly perfect equality between the supply and consumption of the water.

In cases when the pressure upon a sluice is not great, the float *n* may be directly connected with the lever which works the sluice. Fig. 41 represents this simple apparatus: *a*, *a* is the transverse section of the conduit, in which the sluice *b* moves vertically, and is connected by an adjustable link with an oscillating beam *c*, jointed to the top of the short pillar *d*. The other extremity of this beam is similarly connected to a hollow wrought-iron float *e*, which is acted upon by the water whose surface is intended to be preserved at the same constant level, and the supply of which is derived from the conduit *a*; if then the surface at *e* rise, the sluice is depressed, and the discharge by the conduit lessened, and *vice versâ*. This arrangement is evidently only suited to an open conduit, in which no great pressure can be brought upon the sluice; if applied to the mouth of a closed pipe with a great head of water pressing on it, the friction in the grooves of the sluice-frame would be so great as to require an enormous float *e*, and the action could not fail to be of an irregular character.

The double-beat valve, invented by Hornblower (*vide* Pole on the Cornish Engine, pp. 85–88), is represented fully opened in the transverse section in Fig. 42, and isometrically in Fig. 43. It is intended that the water or steam should pass from A to B when the valve is opened, and that the communication between them be intercepted when it is shut. The shadings sloping to the left represent the movable parts; those to the right indicate the fixed parts of the valve. The value of its peculiar construction may be best appreciated by considering the tests of a good valve, which should, in the first place, evidently afford a large passage to the steam or water, with a small displacement; and, secondly, should be capable of being opened with a small force. These conditions are admirably fulfilled in the double-beat valve, which consists of a fixed part or seat C, formed by five partitions, which radiate from a central axis, and are joined below to a ring, *a*, and closed on top by a circular disc, in one piece with the partitions, and covering the spaces between them, and also by a movable part, D, the valve proper, which is a sort of case surrounding the seat C, and having a vertical motion, sliding up and down the exterior edges of the partitions in C; this case is open on the top, and connected with its actuating rod *n*, by the arms *r*, *r*.

When it is at the lowest point of its stroke, and shut, it bears upon the bevilled or conical surfaces *a* and *a'*, which have but a very small breadth; when, on the contrary, it is raised, as in Fig. 42, it permits the passage of the water through the different openings shown by the bent arrows. It is evident that by this arrangement it is not necessary to raise the valve through any great height in order to afford a large passage to the water, thus satisfying the first test mentioned above; on the other hand, the valve D, being pierced on its upper part by a circular opening nearly as great as that on the lower part, the force required to raise it is the excess of the pressure of the water or steam per square inch in A over that in B, multiplied into the difference of of the circular areas above mentioned, this difference being evidently the annulus formed by the sum of the horizontal projections of the upper and lower conical surfaces, *a* and *a'*, shown in Fig. 42, projected down from the transverse section.

If this valve or case D had been a simple disc with bevilled edges, as in Fig. 44, we should have required to lift or start it a force equal to the excess of pressure in A over that in B, multiplied into the whole circular area of the top of the disc *v*, *v*; and this would not only have to be provided by the prime mover, but a very greatly increased size and strength given to the rods, joints, &c., which actuate the valve. In a large disc valve, as, suppose, 12 inches diameter, the area being 113.1 sq. inches, and with an excess of pressure in A above that in B of 15 lbs. per square inch, it would require a force of 1696.5 lbs. to lift it. If in an equal double-beat valve each annulus was ¾ inch broad in the horizontal projection, the sum of their areas would be $(12^2 - 9^2) \times 0.7854 = 49.48$ sq. inches; thus the force required for the starting of such a double-beat valve is less than half that necessary for an equal disc-valve, being 49.48 × 15 = 742 lbs., or 954 lbs. in favour of the double-beat valve, and so in proportion for pressures other than 15 lbs.

In the particular case of the valves raised by the float *n*, Fig. 38, it may be, moreover, remarked, that the force necessary to raise them has to be applied but for a very short time, the instant it is raised, the pressure on each side is brought to a state nearly that of equilibrium; the less, then, the resistance to the float at the moment of raising the valve is, the more sensitive it becomes to any alteration in the surface of the water in *l l*, with an absence of any irregular or jerking motion. The flap-valve *a* is consequently retained more steadily at its proper adjustment.

The moderateur lamp affords a most ingenious example, though on a very small scale, of a constant flow of the oil, though the "head," or pressure, varies widely. The annular wick, or Argand burner, is placed on the upper part of the lamp, and is fed with oil from a cylinder which is placed at the lower part of it, and closed at the bottom. The oil is raised from this by the descent of a piston, forced down by the uncoiling of a spiral spring which is compressed in winding up the piston from the bottom of its course after the former time of use; an ascending pipe, passing through this, conveys the oil up to the wick. Now, not only is the spring weaker as it expands with the descent of the piston, but the vertical height it has to raise the oil also increases: thus, if it were not for the contrivance about to be described, we should have the brilliancy of the light continually lessening as the rate of supply of oil to the wick diminished. This difficulty the celebrated James Watt did not quite surmount when he turned his attention to this subject (*vide* Life, pp. 462–465).

Fig. 64 is a vertical section of a lamp at full size. The ascending pipe *a*, *b*, *c*, *d* consists of two parts, one of which *c d* slides within the other. The inner and lower is fixed to the piston, which it passes through, rising with it, and descending also, under the action of the compressed spring. The upper, *a b*, on the contrary, rests fixed, and serves as a sheath to the movable and lower, having a leather packing, shown at *b*, on the interior surface of which *c d* slides in descending with the piston.

A straight wire, or rod, shown separately (Fig. 68), is placed concentrically within the ascending pipe *c d* of a diameter but little less than it at its upper and thickest part, and long enough to enter the movable pipe *c d* when at its lowest position, as in Fig. 64, the lower part; from *z* to *n*, Fig. 68, is only a support. The oil in rising is compelled to pass through the narrow annular space, Fig. 66, between the interior of the movable pipe attached to the piston, and the rod (or *moderateur*); from this it results that it meets with a resistance which causes its upward movement to be very slow. Now as the movable pipe and piston descend, the same length of the *moderateur* is not always engaged in the pipe: at first, when the spring is strongest, and the height that the oil has to rise is least, then also the annular passage is longest, and the resistance to the ascent of the oil greatest; and again, when the piston has descended, and consequently the spring is weaker, and the height the oil has to be raised greater, so, also, the length of this annular space is less, and the resistance to the ascent of the oil diminished in proportion as the ascending force itself is diminished. By a tentative process in each particular case,—filing a portion at the flat surfaces, Figs. 65, 66,—the needle is adjusted so as to give a uniform supply of oil, and make the lamp burn with equable light as long as the spring acts. This principle is evidently applicable to the discharge of water by simple modifications.

108. *Examples on Weirs.*—Let us, as in p. 85, exhibit the effect, on the quantity discharged per second, of the different values of m, namely, 0.60, 0.665, and between 0.662 and 0.595.—*Vide* § 77, pp. 67–68.

Thus, suppose an overfall of 1 ft. in width, having a depth of 1 ft. passing over: required the discharge in one second; the formula $\frac{2}{3} mlH\sqrt{2gH}$ then becomes $\frac{2}{3} \times m \times 8.024$, or $m \times 5.35$.

1. $m =$ (§ 64, p. 52) 0.60, value of $Q = 3.21$ cb. ft.
2. $m =$ (§ 64, p. 52) 0.665 ,, $= 3.558$
3. $m =$ (§ 82, p. 72) 0.66654 ,, $= 3.566$
4. $m =$ (§ 64, p. 54) 0.595 ,, $= 3.183$

In the following questions it is intended to show the effect of the function of the head or charge $H\sqrt{H}$, which occurs in the formula for the discharge over weirs. A certain length is taken, and the discharge with a given head determined, and then this discharge being increased by a given quantity (XXV.), the corresponding increase of H is determined. In the same way, the discharge being doubled, it is sought (XXVI.) to determine the relative increase of the value of H.

(XXIV.) Calculate the discharge over a weir 1100 ft. long, the depth from the surface of still water to the crest of the weir being 0.75 ft., using 0.665 for the value of m, as given in the second case, p. 107, we have $8.024 \times \frac{2}{3} \times 0.665 = 3.558$, and as $\sqrt{0.75} = 0.866$. Hence $3.558 \times 1100 \times 0.75 \times 0.866 = 2541.3$ cb. ft. per sec. In Beardmore's Table II. we find the discharge for 1 ft. of length of weir with 0.75 ft. head, 138.88 cb. ft., and this multiplied by 1100, gives 152,768; but as all his Tables are calculated for the discharge per minute, dividing 152768 by 60, we obtain 2546.13, differing from that calculated above by 4.83; the coefficient used by Beardmore being 0.6665 (§ 82, p. 72), giving 3.566, instead of 3.558 used above.

(XXV.) To what height upon the crest would the water rise if the discharge was increased to 3000 cb. ft. per sec.? We have from these data,

$$3.557 \times 1100 \sqrt{H^3} = 3000.$$

Hence

$$H = \sqrt[3]{\left(\frac{3000}{3.558 \times 1100}\right)^2} = \sqrt[3]{(0.766)^2} = \sqrt[3]{0.587} = 0.837,$$

being an increase of $0.837 - 0.75 = 0.087$ ft., the increase of H being only 11.6 per cent., and that of Q being 17.9 per cent.

The least laborious method of finding cube roots, when no table of logarithms is at hand, is the following:—Assume a number whose cube is nearly equal to the given number; then, as twice this cube, plus the given number, is to twice the given number, plus the assumed cube, so is the assumed root to the true; in this case, for $\sqrt[3]{0.587}$ first assume 0.8, which gives 0.512; secondly, assume 0.84, which gives 0.593. Hence

$$\overline{2 \times 0.593 + 0.587} : \overline{2 \times 0.587 + 0.593} :: 0.84 :: 0.837.$$

And by logarithms we have log. of $0.587 = \bar{1}.7686381$, which, divided by 3, gives $3)\ \underline{\bar{3}.7686381}$

$\bar{1}.92287 93$ answering to 0.83728.

(XXVI.) If the discharge in (XXIV.) had been doubled, calculate the depth of water flowing over the crest. The average discharge in (XXIV.) being doubled, gives $(2 \times 2544 =)$ 5088 cb. ft. per second on a length of 1100 ft. Hence

$$H = \sqrt[3]{\left(\frac{5088}{3.558 \times 1100}\right)^2} = \sqrt[3]{1.69}, \text{ and}$$

log. of 1.69 = 0.2278867, which, divided by 3, gives 0.0759622, answering to 1.19131; deducting 0.75, we have 0.44 ft. for the rise, to be added to the first supposed 0.75, in order to obtain a double discharge, so that, instead of 1.50, i. e. twice the original head, we have but 1.19 ft. on the crest of the weir for twice the original discharge; it is, in fact, evident that 0.75 is multiplied by $\sqrt[3]{2^2} = 1.5866$, instead of by 2.

If the length of the weir in (xxiv.) had been reduced one-half, namely, to 550 ft., calculate the head to which the water would rise upon the crest, the discharge being the same, namely, 2544 cb. ft. per sec. We have now

$$Q = 2544 = 3.558 \times 550 + H^{\frac{3}{2}}.$$

Hence

$$H = \left(\frac{2544}{1956.9}\right)^{\frac{2}{3}} = 1.191 \text{ ft.}$$

(xxvii.) The construction of the weir at Killaloe (" Selection of Specifications") has the peculiarity of not being level on part of the crest. The inclination being 1 in 214, and the rise 1.5 ft., the length with that slope must be 1.5 × 214 = 321 ft.; we have therefore, as the weir is 1100 ft. long, 779 ft. for the level portion, and 321 ft. at an inclination of 1 in 214. Calculate the total quantity discharged over this weir when the depth of water on the level part is 1.8 ft., so as to have 0.3 ft. on the highest part of the crest at the west abutment. If then we divide this sloping part into eight lengths, of 40 ft. each, and calculate the discharge over each length with a head equal to the arithmetic mean of the head at each extremity of the 40 ft. lengths, the discharges will be sufficiently near the truth. The increase of depth on each 40 ft. is evidently $\frac{40}{214}$ ft., equal to 0.18691 ft., and as the depth over the highest point at the west abutment is, by the terms of the question, 0.3 ft., the mean depth for the first 40 ft. is

$$\frac{0.3 + \overline{0.3 + 0.18691}}{2} = 0.393455 \text{ ft.};$$

to obtain the second, third, &c., we have but to add to this successively 0.18691, and consequently obtain the following numbers:—0.393, 0.580, 0.767, 0.954, 1.141, 1.328, 1.702, 1.795; which, being multiplied by their respective square roots, give 0.2468, 0.442, 0.672, 0.932, 1.219, 1.530, 1.864, 2.220.

Hence the eight several discharges through the 40 ft. lengths are found by multiplying the common part of the formula (§ 55) $\frac{2}{3}m, l, H\sqrt{H} . \sqrt{2g}$, that is, $3.558 \times 40 = 142.3$ into the values of $H\sqrt{H}$ given above, and, adding these, we have the total discharge over the sloping part of this weir 1299 cb. ft. per sec. And for the length of 780 ft. of level crest with 1.8 ft. head, we have 6700 cb. ft. per sec. Hence the total discharge is 7999 cb. ft. per sec. As $8 \times 40 = 320$ ft., and the length of sloping portion is 321 ft., we must add one foot to 779, the length of the level portion.

(xxviii.) In the weirs on the Shannon constructed by the Commissioners, it was requisite that salmon-gaps should be constructed, so that the fish be able to migrate up stream at the weirs during such periods as might not afford sufficient depth of water if the whole quantity were uniformly distributed over the total length of the weir. These were 10 feet wide, and the crest 1.5 ft. below that of the weir. Calculate the quantity flowing down three of these salmon-gaps, the water on the level part of the crest being 0.6 ft. deep. Here

$$H = 1.5 + 0.6 = 2.1, \text{ and}$$

$$3Q = 3 \times 3.558 \times 10 \times 2.1 \sqrt{21} = 324.8 \text{ cb. ft.}$$

(xxix.) A feeder or water-course along the side of a valley is required to be augmented by the streams and springs above its level. It is required to determine their total volume. For this purpose the several courses are dammed up at convenient and suitable places, and a narrow board provided, in which is cut an opening for the overfall 1 ft. long, and 0.5 feet deep; it being reasonably surmised that this would be sufficient to gauge the largest of the streams; and another piece was prepared that, when attached to the former, would reduce the length to 0.5 ft. for the smaller. Calculate the total quantity delivered by the five following streams and springs:

No. 1, on being dammed up, flowed over the 1 ft. opening 0.37 ft. deep. Hence $Q = 3.558 \times 0.37 \sqrt{37} = 0.8$ cb. ft.

No. 2, at 0.5 ft. in length of overfall, rose to 0.41 ft. in depth. Hence $Q = 3.558 \times 0.5 \times 0.41 \sqrt{0.41} = 0.467$ cb. ft. per second.

No. 3, at 1 ft. length, was 0.29 ft. high on the overfall, and

$$Q = 3.558 \times 0.29 \sqrt{0\ 29} = 0.555 \text{ cb. ft. per sec.}$$

No. 4, at 0.5 in length, rose 0.19 ft., $0.5 \times 3.21 \times 0.19\sqrt{0.19} = 0.133$ cb. ft. per sec.

No. 5, being a small spring, was not measured by the overfall; but being banked up, a pipe, 0.0416 ft. in diameter, was let through the dam, and when the surface had become stationary, and consequently the discharge through the pipe equal to the supply from the spring, it was gauged into a vessel marked for 1 and 2, &c., imperial gallons; the time required to reach the former was 32 seconds. Hence the spring gave 0.005 cb. ft. per sec., as 6.25 gallons make one cubic foot.

The total quantity, therefore, received by the aqueduct from the lateral springs and streams above its level amounted to 1.96 cb. ft. per second.

(xxx.) On the Manchester water-works weirs are constructed across some of the lateral mountain streams which supply the reservoirs, so that the higher velocity which the water has when flowing over at the greater depths may separate the turbid water, unfit for the town supply, from the clear. Figs. 45, 45*a*, and 45*b* give diagram transverse sections of this weir. In heavy or sudden rains these streams bring down very rapidly water discoloured by peat and earth, and unfit for domestic use; but in fine weather the quantity is much reduced, and the water clear and suitable for the mains of the town. Fig. 45 represents a transverse section of the watercourse which is carried through the masonry of the weir, conveying clear water from other streams, across the valley in which the weir is placed, and so serving as an aqueduct; at the top this is open, as at *m n*, and when the water flows over at a small depth, that is, when it is clear, it falls into the channel, Fig. 45*a*, and is conveyed by it eventually into the main which supplies the town; but if it rise and discharge a greater body of water, the increased velocity projects it beyond the edge of the opening, Fig. 45*b*, and it thus passes over the longitudinal opening, and flows down to the compensation reservoir for the supply of water to the mills situated on the river.

By referring to § 8, we find the means of calculating the curve of any issuing jet of water. But in this case we have a different velocity, and therefore a different parabola for every lamina into which we may suppose the water divided. Fig. 46*a* represents the different paths taken by each, that for the mean velocity at $\frac{4}{9}$ths of the depth being drawn in a full line; hence those above will tend to depress the curve, and those below, on the contrary, to carry it more up towards the horizontal line; we may therefore suppose the whole sheet of water to be carried out in a curve at top and bottom parallel to that of the mean velocity, Fig. 46. If therefore we put H_1 for the depth of the water flowing over the weir, the mean velocity being $\frac{2}{3}$rds of that at the bottom, we have $v=\frac{2}{3}\times 8.024\times\sqrt{H_1}$ for this mean velocity, and the curve taken by the lowest lamina is that due to a head $\frac{4}{9}H_1$, for if in the expression (§ 48, p. 35)

$$z'=\frac{4}{9}\left(\frac{H\sqrt{H}-h\sqrt{h}}{H-h}\right)^2,$$

we put $h=0$, the resulting value of z' is $\frac{4}{9}H$. Now in Fig. 45 let $x=1$ ft., and therefore $y=0.83$ ft.; hence, from § 8,

$$\frac{4}{9}H_1=\frac{y^2}{4x}=\frac{0.83^2}{4}=0.1722 \therefore H_1=9\times 0.1722\div 4=0.3874 \text{ ft.}$$

So then, when the water flowing over has a depth at or greater than 0.3874 ft., it is carried completely over the longitudinal opening. We must, then, to gauge the stream in wet seasons, and so proportion x to y that the volume of water, from the head necessary to discharge it, have velocity sufficient to pass over the opening $m\,n$; at lesser depths it strikes against the point, and in part enters the clear water-channel, and in part flows over the weir; for this reason it is necessary to have a cover of timber, that the attendant may turn down upon the opening during such period if, at the commencement or end of a flood, the water should be turbid at such a depth as would not completely pass over the opening $m\,n$.

Calculate at what depth the water all flows in. If we suppose in Fig. 47 that $nr=H_1$, which we may do, though it be not normal to the axis of the sheet of water, then $y+H_1=0.83$ and $H_1=0.83-y$, also $\frac{4}{9}H_1=\frac{y^2}{4}$ in this substitute for H_1 its value above, we have

$$\frac{4}{9}(0.83-y)=\frac{y^2}{4}\text{ and }\frac{4}{9}\times 0.83=\frac{y^2}{4}+\frac{4}{9}y;$$

$$\text{or }\sqrt{2.266}=y+\frac{8}{9},\text{ or }1.5-0.888'=y;$$

$$\text{hence }y=0.612\text{ and }H_1=0.833-0.612=0.221\text{ ft.}$$

If, then, we observe in ordinary seasons a stream discharging 26.6 cb. feet per sec., the water being clear, and the most convenient length of the crest of the overfall being 60 ft.,—we may, having selected some convenient depth $n_1\,m$, so adjust the opening mn that the whole of the clear water may fall into it. As a first step, we must calculate H_1, for the length 60 feet, and discharge 26.6, if the coefficient m be taken equal to 0.6665 (§ 82); we have, therefore, $H_1 = \sqrt[3]{\left(\frac{26.6}{3.556 \times 60}\right)^2}$ $= 0.25$ (§ 71), and the vertical depth of n (Fig. 47) below the crest at m being given, we may calculate, first, the value of y, that is $n_1\,n$, so that the curve of the *upper* surface of the sheet of water flowing over the crest may fall within the point n, and the whole stream be carried down the clear water aqueduct. Let mn_1 be taken at 1 ft., then from

$$y^2 = \frac{4v^2x}{2g} \text{ (§ 38), we have } y = \frac{2v\sqrt{x}}{\sqrt{2g}}, \text{ and}$$

substituting for v its value in this particular case where $H_1 = 0.25$, we have $v = \frac{2}{3}\sqrt{2gH_1} = \frac{2}{3} \times 8.024\sqrt{0.25}$ (§ 48), and also for x its value, which is $mn_1 + H_1 = 1.25$ ft. Hence

$$y = \frac{2 \times \frac{2}{3} \times 8.024\sqrt{0.3125}}{8.024} = \frac{4}{3} \times 0.559 = 0.745 \text{ ft.}$$

And secondly, we may calculate at what amount of discharge and head H_1 the curve of the *lower* parabola of the sheet of water will pass completely over the opening mn, and so the stream, now turbid, be all carried over the clear water aqueduct into the settling and compensation reservoirs. Call the sought depth D, and as x is now 1 ft,, we have $y = 0.745 = \frac{2 \times \frac{2}{3} \times 8.024\sqrt{D}}{8.024}$ $= \frac{4}{3}\sqrt{D}$ and $D = \left(\frac{3}{4}0.745\right)^2 = 0.31248$, the discharge being about 37 cb. ft. per sec.

(XXXI.) To determine generally the relation between the length and depth of weir having the same discharge, put

$$\frac{2}{3} \times m \times l_1 \times h_1\sqrt{h_1} = \frac{2}{3} m \times l_2 \times h_2\sqrt{h_2};$$

hence

$$h_1^{\frac{3}{2}} : h_2^{\frac{3}{2}} : l_2 :: l_1,$$

and

$$h_1 : h_2 :: l_2^{\frac{2}{3}} : l_1^{\frac{2}{3}},$$

$$\therefore h_1 = h_2\left(\frac{l_2}{l_1}\right)^{\frac{2}{3}},$$

$$\log h_1 = \log h_2 + \frac{2}{3}(\log l_2 - \log l_1).$$

Calculate the height to which the water upon a weir 545 ft. long will rise when it is flowing down from another weir higher up upon the same river, whose length is 750 ft., and on which it rises 0.68 ft., it being supposed that no additional supply has been received in the intervening part of the course.

$$\begin{array}{rl}
\text{Here} \quad \log l_2 = & 2.8750613 \\
\log l_1 = & \underline{2.7363965} \\
& 0.1386648 \times \frac{2}{3} = 0.0924432 \\
\text{and } \log h_2 = & \bar{1}.8325089 \\
& \underline{0.0924432} \\
& \bar{1}.9249521 \text{ and } h_1 = 0.84 \text{ ft.}
\end{array}$$

By the above method, namely, by discharging the same quantity of water over weirs of different length and measuring the depths, may be determined experimentally the value of the index of H to which the discharge is proportional, on the supposition that m is constant, and that the discharge is directly as the length, for then

$$Q = \tfrac{2}{3} \,.\, m \,.\, l_1 \,.\, h_1^a = \tfrac{2}{3} \,.\, m \,.\, l_2 \,.\, h_2^a,$$

and therefore

$$\left(\frac{h_1}{h_2}\right)^a = \frac{l_2}{l_1}, \text{ or}$$

$$a\,(\log h_1 - \log h_2) = \log l_2 - \log l_1,$$

and

$$a = \frac{\log l_2 - \log l_1}{\log h_1 - \log h_2}.$$

The following Table is arranged from Series VII., Table X. of J. B. Francis, "Lowell Experiments," already mentioned, § 61, p. 47 :—

TABLE *showing the results of Experiments to determine the Index of H.*

	Total length of weir in feet.	Or, very nearly,	Depth on crest of weir in feet.	Average index.
1	16.980	17.0	0.51837	1.478
2	13.978	14.0	0.59514	
3	10.489	10.5	0.72733	
4	8.489	8.5	0.83614	
5	6.987	7.0	0.95882	
6	5.487	5.5	1.13087	

(XXXII.) In the construction of reservoirs it is necessary to have a weir whose crest is on the level of the intended top-water line, with reference to which line the height of the embankment and of the puddle-wall must be designed. The length of this weir must be such that the water of a maximum rain-fall shall not rise above a certain height. We may take the greatest rain-fall at 2 inches in 24 hours; this depth must be multiplied into the area of district which drains into the reservoir. We thus have, first, the total volume of water; and secondly, supposing the rain to have fallen at a uniform rate during the 24 hours, or at least to have been delivered by the water-courses into the reservoir at a uniform rate, we thence obtain the quantity per minute or per second which this weir must discharge. We then assign a certain depth upon the crest, to which the water must be limited, and consequently from the depth H and discharge Q we obtain L.

Thus, suppose the area of the rain-basin or district draining into the reservoir were 6536 acres, and the maximum depth of rain in 24 hours to be 2 inches, we reduce both to the same unit of feet. The acre contains 10 square chains of 66 feet each, $66^2 \times 10 = 43560$ sq. ft., and 2 inches $= 0.1666'$ ft.: hence, $43560 \times 6536 \times 0.1666' = 47451360$ cb. ft. in 24 hours, which, reducing to seconds, we have $24 \times 60 \times 60 = 86400$; and dividing $47451360 \div 86400 = 549.2$ cb. ft. per second, entering the reservoir, the length of weir to discharge this with a rise on the crest of 1.5 ft. is found

$$\frac{549.2}{\frac{2}{3} \times 0.66 \times 1.5\sqrt{1.5} \times 8.024} = \frac{549.2}{6.48} = 84.75 \text{ ft.}$$

As, however, the sluices for discharging the storage would be opened, the rise upon the crest could be readily kept down to 1 ft.

EXAMPLES ON CHAPTER II.

FLOW OF WATER UNDER A VARIABLE HEAD.

109. In § 88 we have the formula—

$$T = \frac{2A\sqrt{H}}{mS\sqrt{2g}}.$$

(XXXIII.) This has been made use of by Dr. Young to determine the value of *m*. A tube 1 inch in diameter is filled for 9 inches with mercury; at the bottom is an orifice $\frac{1}{20}$ inch in diameter; the observed time of its total discharge was 140 seconds. Solving the above equation for *m*, we have—

$$m = \frac{2A\sqrt{H}}{T.S.\sqrt{2g}}.$$

Changing the measures from inches into feet, we have—

$2A = 0.083^2 \times 0.7854 \times 2 = 0.0109$ sq. ft. and $\sqrt{0.75} = 0.866$ ft.

$$S = \frac{1}{400} \times 0.00545 = 0.0000136 \text{ sq. ft.}$$

So that—

$$m = \frac{0.0109 \times 0.866}{140 \times 0.0000136 \times 8.024} = \frac{0.0094394}{0.0152777} = 0.62.$$

However, on account of the circular or vortex motion of the fluid at very small depths, no formulæ which give the time for *complete* exhaustion are quite exact. Mercury is, probably, less affected by this motion than water, with which a funnel-shaped vortex is formed over the orifice; this drawing in the air renders the discharge irregular, and reduces the orifice, so that the formula for *partial* exhaustion—

$$t = \frac{2A(\sqrt{H} - \sqrt{h})}{mS\sqrt{2g}},$$

in § 89, may be considered to give more exacts results. The same author has used it also, as in the following experiment.

(XXXIV.) A prismatic vessel, having a diameter of 5.747 inches, has an orifice 0.2 inch at the bottom, and its surface is observed to sink from 16 inches to 1 foot of depth in 53 seconds. Transposing as before, we have—

$$m = \frac{2A\ (\sqrt{H} - \sqrt{h})}{tS\sqrt{2g}},$$

H being 1.33′ ft., and $h = 1$ ft., the value of $(\sqrt{1.33} - \sqrt{1})$ is $1.153 - 1 = 0.153$ ft. The diameter of the vessel being 5.747 inches, or 0.4783 ft., the value of A will be $0.4783^2 \times 0.7854 = 0.1797$ sq. ft.; also $S = 0.0166^2 \times 0.7854 = 0.000218$ sq. ft. Hence—

$$m = \frac{2 \times 0.1797 \times (1.153 - 1 =) 0.153}{53 \times 0.000218 \times 8.024} = \frac{0.055}{0.0927} = 0.60,\ q.p.$$

(xxxv.) A prismatic basin, whose horizontal section is a square of 3 ft. in the side, has at the bottom an orifice 0.09 ft. in diameter; it is filled up to a depth of 6 ft. above the centre of the orifice. Calculate the *time* required for the surface to descend 3.5 ft., counting from the moment of opening the orifice. Here $A = 3 \times 3 = 9$ sq. ft., $S = 0.09^2 \times 0.7854 = 0.00636$ sq. ft.; $H = 6$, and $h = 6 - 3.5 = 2.5$, m being 0.61; therefore from the formula—

$$t = \frac{2 \times 9\ (2.449 - 1.581)}{0.61 \times 0.00636 \times 8.024} = \frac{15.624}{0.03113} = 502'' = 8'\,22''.$$

(xxxvi.) With the same dimensions calculate the time required for the surface to descend 2 ft. Here $h = 6 - 2 = 4$, and $\sqrt{H} - \sqrt{h} = 0.449$ ft.; therefore—

$$t = \frac{18 \times 0.449}{0.03113} = 259.6 = 4'\,20''.$$

(xxxvii.) Again, suppose the descent of the surface to be 5 ft., calculate the time, $h = 6 - 5 = 1$, and $\sqrt{H} - \sqrt{h} = 1.449$, so that—

$$t = \frac{18 \times 1.449}{0.03113} = \frac{26.082}{0.03113} = 837''.84 = 13'\,58''.$$

(xxxviii.) § 91. *Mean hydraulic charge.*—Let us suppose in any prismatic vessel receiving no supply, that the head, at the instant of opening the orifice of discharge, was 6 ft. = H, and at closing it had decreased to 5 ft. = h, calculate the mean constant charge at which, in the same time, the orifice would discharge the same volume of water; the vessel being now, necessarily, supposed to receive that same constant quantity which it discharges with a uniform velocity.

The formula is—

$$H'=\left(\frac{H-h}{2(\sqrt{H}-\sqrt{h})}\right)^2=\left(\frac{6-5}{2\,(2.449-2.236)}\right)^2=\left(\frac{1}{0.426}\right)^2=5.508\text{ft.}$$

If h be taken equal to 4, then $H'=4.96$; if equal to 3, $H'=4.376$; if $h=2$, then $H'=3.732$; and when $h=0$, we have $H'=1.5$.

If in 10″ we observe the surface to fall 2 ft., determine the coefficient of discharge.

If $A=6$ ft., $S=0.01$, and $T=10''$, then H being $=6$, and $h=4$, we have $Q'=12$ cb. ft., and $Q=1.2$, and $H'=2.227$. Hence—

$$m=\frac{1.2}{0.1\times 8.024\times 2.227}=\frac{1.2}{1.787}=0.67.$$

(XXXIX.) § 92, p. 67. A reservoir, half an acre in area, with sides nearly vertical, so that it may be considered prismatic, receiving a stream which yields 9 cb. ft. per second, discharges through a sluice 4 ft. wide, which is raised 2 ft.; calculate the time required to lower the surface 5 ft., the charge upon the centre of the sluice, when opened, being 10 ft. From the formula given at the end of § 92, we have, substituting the numerical values, $A=21780$ sq. ft. the acre, being 43560 sq. ft.; $S=8$ sq. ft., m being found 0.70, and $h=10-5=5$, also $q=9$ sq. ft. per second,

$$t=\frac{2\times 21780}{(0.7\times 8\times 8.024)^2}\Big\{0.7\times 8\times 8.024\,(\sqrt{10}-\sqrt{5})$$
$$+\,2.303\times 9\times\log\frac{0.7\times 8\times 8.024\,\sqrt{10}-9}{0.7\times 8\times 8.024\,\sqrt{5}-9}\Big\}.$$

In this we have $0.7\times 8\times 8.024=44.9$, and $\sqrt{10}-\sqrt{5}=3.162-2.236=0.926$. Hence—

$$t=\frac{43560}{2016}\{44.9\times 0.926+20.7\log 1.455\}$$
$$=21.607\{41.6+3.37\}=972''=16',\ 12''.$$

If q, the constant supply received by the reservoir, had been 20 cb. ft. per second, then—

$$\frac{(44.9\times 3.162)-20}{(44.9\times 2.236)-20}=\frac{121.97}{80.40}=1.517,$$

the log. of which is 0.1809856 (in the former case subtracting 9 we had $\frac{132.97}{90.4} = 1.455$, the log. being 0.1628630), and the value of t is now $21.607\{41.6 + 2.303 \times 20 \times 0.181\} = 1079'' = 17' 59''$ to lower the surface 5 ft.

(XL.) Referring to the latter part of § 92, in order to determine the depth which the surface would descend in a given interval of time, the formula must be arranged so as to separate the factors of $\sqrt{H}$ from $\sqrt{h}$, then transposing, so as to make the left-hand side = 0, we have

$$t - \frac{2A}{(mS\sqrt{2g})^2}\{mS\sqrt{2g}\sqrt{H} + 2.303 \times q \times \log(mS\sqrt{2g}\sqrt{H} - q))\}$$

$$+ \frac{2A}{(mS\sqrt{2g})^2}\{mS\sqrt{2g}\sqrt{h} + 2.303 \times q \times \log.(mS\sqrt{2g}\sqrt{h} - q\} = 0.$$

Let us suppose all the letters to have their former values, t being taken at 20 minutes, calculate the value of h—

$$(t =)\ 1200'' - \frac{43560}{2016}\{44.9 \times 3.162 + 20.73 \times \log 133\} =$$
$$1200 - 4020 = -2820,$$

aud thus we have—

$$21.61 \times \{44.9\sqrt{h} + 20.73 \times \log(44.9\sqrt{h} - 9)\} - 2820 = 0,$$

when the true value of h is substituted. To further prepare this last expression for the tentative determination of h, we must multiply out by 21.61, hence—

$$970.3\sqrt{h} + 448 \log(44.9\sqrt{h} - 9) - 2820 = 0.$$

If we take at first—

$\sqrt{h} = 2$, the equation becomes $-\ 25 = 0$,
$\sqrt{h} = 2.4$ „ „ $+\ 422 = 0$,
$\sqrt{h} = 2.01$ „ „ $-\ 14 = 0$,
$\sqrt{h} = 2.1$ „ „ $+\ 82.7 = 0$,
$\sqrt{h} = 2.03$ „ „ $+\ 7.44 = 0$,
$\sqrt{h} = 2.023$ „ „ $-\ 0.1 = 0$, and $h = 4.09$.

The surface, therefore, descends 5.9 feet in 20′.

(XLI.) § 93. A pond, whose area is 12000 square feet, has an overfall outlet 3 feet wide, which at the commencement of the discharge has a head of 2.8 feet; calculate the length of time required for the surface to descend 1 foot, it being supposed that no supply is received.

We have then $H = 2.8$, and $h = 2.8 - 1 = 1.8$, the value of m being taken at 0.61.

The formula—

$$t = \frac{3A}{ml\sqrt{2g}}\left(\frac{1}{\sqrt{h}} - \frac{1}{\sqrt{H}}\right)$$

being put into numbers for this question; we have—

$$t = \frac{3 \times 12000}{0.61 \times 3 \times 8.024}\left(\frac{1}{\sqrt{1.8}} - \frac{1}{\sqrt{2.8}}\right) = 2452\left(\frac{1}{1.34} - \frac{1}{1.673}\right)$$

$$= \frac{2452}{1.34} - \frac{2452}{1.673} = 1830 - 1466 = 364'' = 6'\ 4''.$$

Calculate the time in which the surface descends 0.5 feet. In this case $h = 2.8 - 0.5 = 2.3$, and $\frac{1}{\sqrt{2.3}} = \frac{1}{1.516}$. Hence—

$$\frac{2452}{1.516} - \frac{2452}{1.673} = 1617 - 1466 = 151'' = 2'\ 31''.$$

Again, if we suppose the depth descended to be 1.5, and all the other quantities remain the same, we shall thus have $h = 2.8 - 1.5 = 1.3$, and $\frac{1}{\sqrt{1.3}} = \frac{1}{1.14}$, so that

$$t = \frac{2452}{1.14} - \frac{2452}{1.673} = 2151 - 1466 = 685'' = 11'\ 25'';$$

the depths then being 0.5, 1, 1.5 feet; the corresponding intervals are 2′ 31″, 6′ 4″, 11′ 5″. If $h = 0$, it is evident that t becomes infinite, as $\frac{2452}{0} =$ infinity, and so also of any finite number in the numerator, arising from any other data. If the depth sunk had been nearly equal to the whole charge at the commencement, as, suppose 2.4, so that $h = 2.8 - 2.4 = 0.4$, then $\frac{1}{\sqrt{0.4}} = \frac{1}{0.6324}$, and

$$t = \frac{2452}{c.6324} - \frac{2452}{1.673} = 3877 - 1466 = 2411'' = 40'\ 11''.$$

(XLII.) § 95. In question XIII., page 90, Fig. 31, taken from D'Aubuisson, the time of filling the lower part of a canal lock, on the Canal du Midi, is calculated, i. e. up to the level of the centre of the sluices, placed in the upper pair of gates; we can now, by the 2nd case of § 95, calculate the time of filling up to the level of the upper reach, from the centre of the sluice doors, which, added to the 25″, as determined in XIII., will give the total time. Substituting in the formula—

$$T = \frac{2A}{mS\sqrt{2g}} \times \sqrt{H},$$

the several numerical values given at p. 91, we shall have—

$$T = \frac{2 \times 3503.6}{0.548 \times 13.532 \times 8.024} \times \sqrt{6.3945},$$

that is

$$\frac{7007.2}{59.5} \times 2.53 = 298 = 4' \ 58'',$$

to which adding 25″, we have 5′ 23″ as the total time of filling a lock of such dimensions.

(XLIII.) The locks on the Montgomeryshire canal have a length of 81 and width of 7.75 feet; and at one, named the Upper Belun Lock, the lift or rise was 7 ft. A pipe leads the water from the upper level, and discharges below the surface of the lower level in the lock chamber, the diameter of which is 2 feet. As the mouth of this pipe is a square, 2 feet in the side, gradually altered into a circular pipe, 2 ft. in diameter, we may take $m = 1$, a result which is justified by comparing the observed time of filling this lock with that calculated by the formula—

$$T = \frac{2A}{mS\sqrt{2g}}\sqrt{H},$$

when m is put equal to unity, for

$$\frac{2 \times 81 \times 7.75}{1 \times 2^2 \times 0.7854 \times 8.024} \times 2.645 = 132'' = 2' \ 12'',$$

the observed time being 2′ 10″.

CHAPTER III.

FLOW OF WATER THROUGH PIPES, ARTIFICIAL CHANNELS, AND RIVERS.

110. Gravity is the sole force that acts upon a mass of water left to itself in a bed of any form; it produces all the motion which takes place,—the inclination of the surface of the water in the channel is the immediate cause of motion, being that which enables gravity to act: and thus the measure of this force is in feet per second, $g \times \sin i$,—in which g represents the measure of the force of gravity at the earth's surface, being the rate of motion at which a body is moving at the end of one second when falling freely in vacuo, or 32.1908 ft.; and i is the number of degrees, &c., of inclination of the surface of the water in the channel to the horizon; and $\sin i$ the ratio of the height fallen in any length to that length, or the fraction $\frac{\text{height}}{\text{length}}$.

Thus, if in one mile the surface was lowered 12 ft., we should have $\sin i = \frac{12}{5280}$, or $\frac{1}{440}$, and the constant dynamic force producing motion is measured by

$$g \sin i = 32.1908 \times \frac{1}{440} = 0.07316 \text{ ft. per sec.}$$

The angle of inclination being that which has the natural sine 0.0022727, or 0° 7′.45″. If, then, water flowing in a channel or pipe, and subject to this constant accelerating force, meet with no resistance, it will descend with an increasing velocity which would never be found uniform.

But observation and experience show that in open channels and pipes, even those of very great inclination, the rate of motion very soon becomes uniform. Bossut made the following experiment to prove this truth directly:—Having constructed a canal in wood, 650 ft. long, with a slope of 1 in 10, and marked off equal spaces of 108 ft. each, it was found that the water traversed each space, except the first, in equal times. There must then exist a retarding force, which destroys at each instant the effect of the accelerating force, and which, when the velocity has become uniform, is necessarily equal to it.

But in pipes, channels, &c., there can be no retarding force but that which arises from the resistance of the sides or bed: and of its existence we cannot doubt, for the simple experiment of the measurement of the discharge through a tube in a certain time, and again when the tube has been lengthened—all else remaining the same—proves that the time required to yield a certain volume of water has been increased also; and this can only arise from the fact that the tube, or other channel, by reason of its increased length, offered a greater resistance to the velocity. The *surface* thus opposed motion.

To these retarding forces the name of Friction has been applied: though, from the difference between the laws of friction of water flowing over its resisting bed, and the friction of solid bodies sliding upon each other, we must look upon it as the application of an old word in a new sense, in preference to adding a new term to express this peculiar resistance. It may be useful to state here briefly the laws of friction of solid bodies, with the view of showing this contrariety.

111. *First Law.*—Experiment has shown that the friction or resistance to motion of bodies, sliding upon their surfaces of contact, is directly proportional to the force or weight pressing the two surfaces together, and differs only with the nature of the sliding surfaces, as wood, brass, iron, &c.

Second Law.—The amount of friction is independent of the extent of the surface pressed, provided the whole amount of the pressure remains the same, and that the substance of the surface pressed is the same.

Third Law.—The friction of a body, when in a state of continuous motion, bears a constant ratio to the pressure upon it, which is the same, whatever may be the velocity of the motion,—it is, in other words, independent of the velocity. Thus the first only of these laws can be expressed algebraically.

112. In the case of fluids, it has been shown that the resistance to motion which we observe, and which has been called friction also, is, on the contrary—

First Law.—Independent of the pressure, that is, that the resistance to motion in a pipe with a head or pressure of, suppose, 100 ft., is the same as if the head were but 50 ft., or any other height. Dubuat had proved this by experiments on the oscillation of water in syphons, which the author of the article "River" (Encyclopædia Britannica) has thus improved upon:—

Two vessels ABCD, *abcd* (Fig. 48), were connected by the bent pipe EFG *gfe*, which turned round in the short tubes E and *e*, without allowing any water to escape; the axis of these tubes being in one right line. The vessels were about ten inches deep, and the branches FG, *fg* of the syphon were about five feet long. The vessels were set on two tables of equal height, and (the hole *e* being stopped) the vessel ABCD, and the whole syphon, were filled with water, and the water was poured into the vessel *abcd* till it stood at a certain height LM. The syphon was then turned into a horizontal position, and the plug drawn out of *e*, and the time carefully noted which the water employed in rising to the level HK*kh* in both vessels. The whole apparatus was now inclined so that the water ran back into ABCD. The syphon was now put in a vertical position, and the experiment repeated: no sensible or regular difference was observed in the time; yet in this experiment the pressure on the part G*g* of the syphon was more than six times greater than before. As it was thought that the friction on this small part (only six inches) was too small a portion of the whole resistance, various additional obstructions were put into this part of the syphon, and it was even lengthened to nine feet; but still no remarkable difference was observed. It was even thought that the times were less when the syphon was vertical; nor has any variation ever been observed in the friction of water upon glass, lead, iron, wood, &c. (Principes d'Hydraulique, tome i., §§ 34 and 36, Dubuat.)

Second Law.—The resistance is, at any one velocity, proportional to the surface exposed to the action of the flowing water. In order to obtain an expression for this law, we may remark, first, that in any channel or pipe the resistance of the surface (Fig. 49) ABCD *abcd* is, from the natural adhesion of the lamina into which we may suppose the water divided, and which couches or lamina are indicated by the dotted lines in the figure, shared by all the particles in the volume of water between the two transverse planes whose distance is Aa,—that nearest the sides being most retarded, and each in succession less and less influenced. The greater, then, that *surface* is, the greater is the resistance. But the greater the *volume* upon which this retarding action of the surface has to act, the less reduced will be the velocity of the first, and therefore of each successive lamina: and thus we have the resistance directly proportional to the surface, and inversely as the volume, i.e. $\frac{\text{area}}{\text{volume}}$; or as the surface is $Aa \times (ab + bc + cd)$, and the volume $Aa \times (\text{area } abcd)$, we may evidently in $\frac{Aa \times (ab + bc + cd)}{Aa \times (\text{area } abcd)}$ strike out from each the length Aa of the channel common to both. The length $ab + bc + cd$ is called the border, or the wetted perimeter: and thus we have the resistance directly proportional to the border, and inversely as the area of the transverse section perpendicular to the axis of the stream. If, then, we put S for the area of section, and C for the contour of the border, we have the resistance proportional to $\frac{C}{S}$.

Third Law.—The resistance is proportional to the square of the velocity nearly, the border being constant. For the *number of particles* drawn in one second from their adhesion to the sides of the channel or pipe is proportional to the number of feet per second with which the water is moving, that is, to the velocity. And the *force* with which they are drawn is also as the same number of feet per second, or the same velocity: and thus the passive resistance of the wetted border to the flow of the water is proportional to the product of the velocity into the velocity; this part, then, of the expression for the resistance is represented by av^2, a being a constant, determined hereafter.

Experimenters have shown that this gives the resistance a very little too high, and that with velocities increased in the ratio 2, 3, 4, &c., it is not represented by $a \times 4$, $a \times 9$, $a \times 16$, &c., but more nearly by adding the simple power of the velocity, thus, $a\,(v^2 + bv)$, the series of numbers $v^2 + v$ not increasing so fast as v^2.

Fourth Law.—In gases and elastic fluids we also have the friction proportional to the specific gravity or density.

In order to obtain from these laws a formula for the discharge of water through pipes and channels, we must make use of the well-known principle, that when any body is moving with a uniform velocity, the accelerating are necessarily equal to the retarding forces: for if the accelerating forces be supposed greater than the retarding, the velocity must increase; and if they should become less, then the velocity must, on the other hand, decrease. Our object must now be, as in the former chapters, to find, first the mean velocity, for this multiplied into the transverse area gives the discharge with a given inclination: and we can thus solve the questions that arise in practice, such as the requisite dimensions of pipe or channel to convey a given quantity of water, &c., &c.

Now in any pipe or channel, whose length is l, and whose height, from the surface of the supply to the point of discharge or extremity of l, is represented by h, we have the accelerating force expressed by $\frac{h}{l} \times g$, or sin inclination of surface into gravity.

The retarding forces are, from the second and third laws above given, neglecting bv, proportional to

$$(a) \quad \dots\dots\dots \quad \frac{C}{S} \times v^2,$$

and therefore we have

$$(b) \quad \dots\dots \quad g \times \frac{h}{l} = v^2 \times a \times \frac{C}{S}.$$

Each side of this equation represents an equal number of feet per second. The left-hand being 32.1908 ft. per sec. reduced, by being multiplied by a fraction whose value depends on the inclination of the surface, that is $\frac{\text{height}}{\text{length}}$. And the right-hand side being the square of the number of feet per second with which, at a mean, the water is moving when the motion has become uniform, reduced by the constant multiplier a, and also by a quantity depending on the figure of the transverse section of the channel,—a being some constant quantity to be determined by experiment. If the formula be correct, all good experiments will give the same value for a, that quantity by which the right-hand side of (b) must be multiplied to produce the equality. We may, however, simplify the expression by dividing out by g, and thus we have

$$(c) \quad \dots\dots \quad \frac{h}{l} = \frac{a}{g} \times \frac{C}{S} \times v^2,$$

and as g is constant, put $\frac{a}{g} = a'$, which must be constant if a be so; solving, then, for a', we have

$$(d) \quad \dots\dots\dots \quad \frac{h}{l} \times \frac{S}{C} \times \frac{1}{v^2} = a'.$$

Substituting the actual data of experiments for the letters of the left-hand side, we obtain the value of a', and comparing different experiments together, we find that it remains very nearly the same in all.

The celebrated Smeaton has given in his Reports (vol. ii. p. 297) a series of experiments on the velocity of water flowing through pipes under pressure. One of these, No. 7, had the following data:—Diameter of pipe, $4\frac{1}{2}$ inches, or 0.375. ft; length, 14637 ft.; fall or head, 51.5 ft.; and $v = 1.815$ ft. Hence $\frac{51.5}{14637} \times \frac{0.375^2 \times 0.7854}{0.375 \times 3.1415} \times \frac{1}{1.815^2}$ = very nearly, 0.0001 = a'.

The quantity discharged is given by Smeaton in Scotch pints, which he states contain 103.4 cb. inches, and therefore, the number of cb. ft. in one pint is $\frac{103.4}{1728} = 0.05984$, and as 200 pints per minute were discharged, we have $Q' = 11.968$ cb. ft. Hence as $\frac{Q'}{tS} = v$, we have $\frac{11.968}{60 \times 0.110247} = 1.815$ ft. per second.

Mr. Provis has published in the "Transactions of Civil Engineers," vol. ii. p. 203, some experiments on the flow of water through pipes $1\frac{1}{2}$ inch = 0.125 ft. in diameter; of these, No. 4, with a length of 100 ft., delivered 2 cb. ft. *per minute*, with a head of 2.5 ft. (It is presumed that the orifice of entry of the water was of the best form.) Here the velocity will be (2 cb. ft. $\div$ $60 \times 0.125^2 \times 0.7854 =$) 2.72 ft. per second: and hence the value of a' is found

$$\frac{2.5 \text{ ft.}}{100 \text{ ft.}} \times \frac{0.125^2 \times 0.7854}{0.125 \times 3.1415} \times \frac{1}{2.72^2} = \text{very nearly, } 0.0001\,;$$

and as from these and many other experiments $a' = 0.0001$, we have $\frac{1}{a'} = 10{,}000$.

Substituting, then, this value of a', and solving the equation (d) for v, we have this expression for the mean velocity

$$v = \sqrt{\frac{h}{l} \times \frac{S}{C} \times 10000}\,;$$

or, taking the root of the factor 10,000, and placing it outside,

$$(e) \quad . \quad . \quad . \quad . \quad . \quad v = 100\sqrt{\frac{h}{l} \times \frac{S}{C}}.$$

From this expression it is evident that many geometrical questions arise in designing the best form of channel, whether circular, rectangular, or trapezoidal, to convey given quantities of water: a given area having, with the same condition as to ratio of slopes, a great number of different borders, and one a minimum, and, *vice versâ*, a given border having a number of different sectional areas, and one a maximum.

The quantity $\frac{S}{C}$ has been called the hydraulic mean depth: it is, in the case of rectangular and trapezoidal sections of channel, represented by a line *de* (Fig. 49); the rectangle under which, and the border $(ab + bc + cd)$, extended into one right line *ad*, is equal to the area of the section; the greater it is, the less the relative resistance of the surface to the volume of water passing over it. It is important, therefore, to have a clear idea of the influence of the figure of the transverse section of the channel upon the magnitude of this quantity, on which, other things being the same, the mean velocity depends; being directly proportional to its square root. As a simple form, let us take a channel whose transverse section is a rectangle, and, first, suppose the *border* to be constant, secondly, the *area*.

Now, when the border is constant, it is evident that there are two extreme positions of the figure: one, when the depth becomes zero, in which case the bottom width must equal the constant border, as suppose 200 ft., or yards, or metres, and coincide with the line of water surface. This is shown in Fig. 69, in which the line ACB is the surface of the water of the several transverse sections, and CD the vertical central line with reference to which they are all symmetrically arranged: A, C, B represents 200 ft. at a scale of 40 ft. to the inch, and is, as has been stated, the limiting figure when the depth becomes zero. The other extreme position of the figure of the rectangular section of a constant wetted border is when the bottom width becomes zero, and then the depth must be equal to half the border, or 100 ft., and coincide with the vertical central line C, D.

Now if we take any other rectangle, as $A'D'D_1B_1$, having the same border, and the vertical central line coinciding with CD, the water-surface line also coinciding with ACB, we shall have $AA' = A'D'$, and $BB_1 = B_1D_1$, and therefore the points D' and D_1 are in the right lines joining the point D and the points A and B: and thus all possible rectangles having this constant border may be inscribed in the isosceles right-angled triangle ADB.

The following Table gives the dimensions of those transverse sections which are drawn in Fig. 69 within the triangle ADB:—

TABLE *showing the value of the Hydraulic Mean Depth, Area, &c., the Channel having a rectangular transverse section, the Border being constant.*

Depth in feet.	Bottom width in feet.	Area in square feet.	Hydraulic mean depth, or area ÷ 200.	Square roots of hydraulic mean depths.
0	200	0	0.00	0.00
1	198	198	0.99	0.99
2	196	392	1.96	1.40
3	194	582	2.91	1.84
5	190	950	4.75	2.18
10	180	1800	9.00	3.00
20	160	3200	16.00	4.00
25	150	3750	18.75	4.33
30	140	4200	21.00	4.58
40	120	4800	24.00	4.90
50	100	5000	25.00	5.00
60	80	4800	24.00	4.90
70	60	4200	21.00	4.58
75	50	3750	18.75	4.33
80	40	3200	16.00	4.00
90	20	1800	9.00	3.00
95	10	950	4.75	2.18
97	6	582	2.91	1.84
98	4	392	1.96	1.40
99	2	198	0.99	0.99
100	0	0	0.00	0.00

It will be perceived at once that the area and hydraulic mean depth increase progressively up to that figure in which the depth is equal to half the bottom width, the rectangle being half of the square whose side is the bottom width: and although the ratio of the hydraulic mean depth to the depth is continually decreasing, yet the former quantity increases until this ratio has become ½, after which point it diminishes, having the same value at depths equidistant from the maximum. The inner curve CND, Fig. 70, gives a diagram representation of the results of the Table. The depths are severally plotted on the line CD from C as zero, at the same scale as in Fig. 69, and the hydraulic mean depths as vertical ordinates at the points in CD, corresponding to the depths. Joining the termination of these ordinates, we have the curve line CND, and by it we can obtain the hydraulic mean depth due to any particular depth by drawing a vertical line up to the curve at the point.

If, instead of a constant border, we assume the rectangular transverse section to have a constant area, and for readier comparison with the foregoing take 5000 sq. ft., that area, namely, which was a maximum with the border of 200 ft., we now find that, with a depth indefinitely small, the bottom width must be indefinitely great; and when we assume a bottom width indefinitely small, then the depth must be indefinitely great, as the product in each case is a given quantity: and rectangles, of figure intermediate between these extremes, being placed, as before, with respect to the lines AB and CD, we shall find the points corresponding to D' and D_1 lie in a curve, well known as the hyperbola, and represented in Fig. 69 by d, D', d', and d_1, D_1, d_2; the points D' and D_1 being common to the rectangle of constant border and constant area.

TABLE *showing the value of the Hydraulic Mean Depths, &c., &c., the Channel having a rectangular transverse section; Area constant.*

Depth.	Bottom width.	Hydraulic mean depth.	Square roots of hydraulic mean depth.	Border.
0	∞	0	0	∞
1	5000	0.9994	0.9997	5002
2	2500	1.997	1.413	2504
3	1666.6	2.989	1.726	1672.6
5	1000	4.95	2.225	1010
10	500	9.604	3.878	520
20	250	17.24	4.152	290
25	200	20	4.472	250
30	166.6	22.065	4.700	226.6
40	125	24.39	4.939	205
50	100	25	5	200
60	83.33	24.564	4.956	203.33
70	71.4	23.65	4.863	211.4
75	66.6	23.084	4.800	216.6
80	62.5	22.47	4.74	222.5
90	55.5	21.23	4.607	235.5
95	52.6	20.61	4.54	242.6
98	51.02	20.24	4.50	247.02
99	50.505	20.12	4.48	248.505
100	50	20	4.472	250
1000	5	2.494	1.58	2005

All possible rectangles having this constant area may be inscribed in the space formed by the two branches of the curve and the right line A, C, B, produced each way indefinitely. The Table gives the dimensions of some of those drawn in Fig. 69 within this space, the sides having dotted lines.

An inspection of this Table shows that when the area of the rectangle is constant, the hydraulic mean depth increases with the increase of the depth, being at corresponding depths somewhat greater than in the former Table, except at the maximum value, which is the same in each; and at this point, as before, the hydraulic mean depth is half the depth. The outer curve CNR, Fig. 70, gives a diagram of the results, the hydraulic mean depths being plotted, as before, at the depths of channel, from which they were calculated. The curve, commencing at zero at the same point C, also passes through a common point N, but from this it diverges, and soon becomes convex to the line CD produced, which it never can reach, the hydraulic mean depth having always a finite value, in the case of a constant area, as long as the depth is finite. In every part this last curve is exterior to that representing the results of a rectangle with constant border, coinciding only at the points C and N.

If we produce MN, so that MN = NO, and draw the line CO, producing it indefinitely, then the ordinates, as mn or $m'n'$, being produced to cut this line in o and o', we have the ratio of the hydraulic mean depth to the depth at each point, taking n and n' either on the inner or outer curve.

It will be proved generally in § 115, that the best form of channel is, whether the transverse section be rectangular, polygonal, or circular, when half the depth of the water at the centre line is equal to the hydraulic mean depth; a proposition which has appeared in the above Tables for the particular numbers chosen, which are mainly intended to illustrate the importance of the figure of the transverse section of a rectangular channel.

Let us suppose that, with the same inclination, we had two rectangular channels of equal transverse area; but in one the depth and bottom width were 5 ft. and 1000 ft. respectively, and in the other 50 ft. and 100 ft., or numbers in that ratio; then, from the second Table, we perceive that the square roots of the hydraulic mean depths are as 2.214 to 5, and therefore the mean velocity, which is proportional to this quantity, is more than double, and the volume of water flowing down, which is the product of the mean velocity into the transverse area, also more than twice as great. When the border is constant, the comparison gives results still wider. Thus, if from the first Table we take rectangular channels, whose depth and bottom width are 10 and 180, and again 50 and 100 respectively, or any numbers in those ratios, we find the square roots of the hydraulic mean depths are as 3 and 5, and, multiplying each into the area, we have the volumes of water carried down as 5400 to 25000, the inclinations being the same in both channels.

We may also gather from these Tables, that in wide rivers and channels in which the depth is small compared with the width, the mean velocity is very nearly proportional to the square root of the depth, for in such cases the hydraulic mean depth is nearly equal to the depth, as in the upper lines of each Table. It is also evident that on each side about the maximum value the mean velocity does not diminish very rapidly; thus in the second Table the rectangles, 40 by 125, and 60 by 83.3, differ but very little in their mean velocity, and therefore in volume discharged from that of 50 by 100. In the first Table, in like manner, the mean velocities, for depths intermediate between 40 and 60, being nearly identical with the maximum at 50, the volumes discharged will only vary from the maximum discharge in proportion to the areas.

In the case of tubes having a uniform circular section, $\frac{S}{C} = \frac{d^2 \times 0.7854}{d \times 3.1415} = \frac{d}{4}$, the formula ($e$) becomes then, in the case of pipes flowing full,

$$(f) \quad . \quad . \quad v = 100\sqrt{\frac{h}{l} \times \frac{d}{4}} = 50\sqrt{\frac{h}{l} \times d} \text{ ft. per sec.}$$

We have seen, in the second law of friction, that each successive couche or lamina, into which we may suppose the fluid in motion to be divided, is less and less retarded from the border towards the centre of this section: the highest velocity being consequently near the centre and in open channels at the surface. The volume of water which traverses the section of which we speak, in one second, is due to these different velocities; and the velocity, the expression for which we have now determined, is that one of these various velocities with which, if the whole section moved as one solid mass, the discharge would be the same: it is then the mean velocity, and is found in any actual experiment by dividing the volume discharged in one second by the section, as has been done in the two experiments used for the determining the value of a'.

In order, then, to determine the discharge by any channel or pipe, for which we have deduced the value of v from the given inclination and hydraulic mean depth, we multiply the expressions (*e*) or (*f*) by the area. Thus from (*e*) we have

$$(g) \quad . \; . \; . \; . \quad Q = S \times 100 \sqrt{\frac{h}{l} \times \frac{S}{C}};$$

or if we put H_y for $\frac{S}{C}$, the hydraulic mean depth,

$$Q = S \times 100 \sqrt{\frac{h}{l} \times H_y}.$$

And again, from (*f*) we have for pipes running full,

$$Q = 0.7854\, d^2 \times 50 \sqrt{\frac{h}{l} \times d},$$

or

$$(h) \quad . \; . \; . \quad Q = 39.27 \sqrt{\frac{h}{l} \times d^5} \text{ cb. ft. per sec.}$$

In many works and reports the discharge is spoken of per minute, instead of per second: and for this unit of time we have $60 \times 39.27 = 2356.2$ as the factor outside; hence

$$Q = 2356 \sqrt{\frac{h}{l} \times d^5}$$

cb. ft. per minute, which may be written thus,

$$2356 \times \frac{\sqrt{d^5}}{\sqrt{\frac{l}{h}}},$$

being the formula used by Beardmore in calculating Table 5, to which reference is again made in the Examples to this Chapter.

Other formulæ for the mean velocity, generally expressed in words, are in use amongst engineers, which may be derived from those we have now given, namely, that the mean velocity of water in any pipe or channel that has attained a uniform velocity is nine-tenths of the square root of the product of twice the fall per mile into the hydraulic mean depth; or sometimes thus expressed, 0.92 into a mean proportional between twice the fall per mile and the hydraulic mean depth.

These, which would not be given in words but to obviate any disadvantage arising from the student meeting with them for the first time, are consequences of equation (e); for the numerator and denominator of the fraction $\frac{h}{l}$ may be replaced by any numbers having the same ratio. If, then, we make $l = 5280$, i. e. the number of feet in a mile: the numerator, which we may call f, expresses the fall per mile thus, $\frac{h}{l} = \frac{f}{5280}$; and from ($e$) we have, therefore,

$$(i) \quad v = 100 \times \sqrt{\frac{f}{5280} \times H_y} = \frac{100}{72.66} \times \sqrt{fH_y} = 1.3\sqrt{fH_y};$$

and as $1.3 = 0.9166\sqrt{2}$, we have, by substituting this value in (i),

$$(j) \quad . \quad . \quad . \quad . \quad . \quad v = 0.9166\sqrt{2fH_y}.$$

113. From the formula (h) for the discharge of pipes running full under pressure, we can, being given any two of the three quantities Q, the inclination $\frac{h}{l}$, and d, determine the other. Let it be required to find the diameter of the pipe, which, with a given inclination, shall convey a given quantity of water. Dividing equation (h) by 39.27, and squaring both sides, we have

$$(k) \quad . \quad . \quad . \quad . \quad . \quad \left(\frac{Q}{39.27}\right)^2 = \frac{h}{l} \times d^5,$$

and dividing by $\frac{h}{l}$, or multiplying both sides by $\frac{l}{h}$, and extracting the fifth root,

$$(l) \quad . \quad . \quad . \quad . \quad . \quad \sqrt[5]{\left(\frac{Q}{39.27}\right)^2 \times \frac{l}{h}} = d.$$

The requisite inclination is found from (k) by dividing both sides by d^5,

$$(m) \quad . \quad . \quad . \quad . \quad . \quad . \quad \left(\frac{Q}{39.27}\right)^2 \times \frac{1}{d^5} = \frac{h}{l};$$

and if we multiply both sides by l, we obtain h: so that if the length the water has to be conveyed be also amongst the data, we obtain the head or pressure necessary to force the given quantity along a pipe of known length and diameter,

$$(n) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \left(\frac{Q}{39.27}\right)^2 \times \frac{l}{d^5} = h.$$

We cannot, however, fully determine the figure of a rectangular or trapezoidal channel from (g); solving (g) for $\frac{S^3}{C}$ we have

$$(o) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \left(\frac{Q}{100}\right)^2 \times \frac{l}{h} = \frac{S^3}{C}.$$

In this we require, in addition, to be given either S or C, and also the ratio of the slopes of the sides if it be a trapezium; moreover, S and C are so related that, with given slopes, there is a maximum value of S to every given value of C; if S exceed this maximum, the solution is impossible.

114. It is found in practice that certain soils, in every excavation for whatever purpose, require a rate of slope in the sides adapted to the degree of cohesion of the ground, to obviate the danger of slips, which occur when they are too steep: this slope of the banks is, therefore, always found amongst the requisite data in the designing of channels.

In order that the side slopes of channels, intended to be permanent, may stand without any masonry or dry stone pitching, they should have a slope between the rates of 1½ horizontal to 1 vertical, and 2 to 1: being made flatter according as the soil has less tenacity. In some cases even 2½ to 1 has been adopted; the half regular hexagon has slopes of 0.58 to 1; in channels for temporary use we may have 1 to 1.

And so also must the velocity be given; and, for the same reason, some kinds of earth being worn away, and the form of channel destroyed, by a rate which carries down the particles of the soil through which it it excavated, a velocity must therefore be assigned within this rate of motion. It has been determined experimentally for many kinds of earth.

The effect of the velocity of the water, in carrying down the particles of the ground through which the channel is excavated, depends jointly upon their tenacity and size. As to the size, we know that the cubical quantities or weights of any similar bodies decrease faster than their superficial areas; and the pressure or force urging a body down stream being, *ceteris paribus*, proportional to the surface, is relatively greater the less the volume; the smaller the particles, therefore, the less is the velocity required to move them. Mr. Beardmore* in Table 3 gives the following statement of the limit of *bottom* velocities in different materials in feet per minute:—

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* Hydraulic Tables, by N. Beardmore.

30 ft.	will not disturb clay with sand and stones.		
40 ft.	will move along coarse sand.		
60 ft.	,,	,,	fine gravel, size of peas.
120 ft.	,,	,,	rounded pebbles, 1 in. diameter.
180 ft.	,,	,,	angular stones, about $1\frac{3}{4}$ in. ditto.

The beds of rivers, protected by aquatic plants, however, bear higher velocities than this Table would assign.

Such being the natural limitations in the choice of any particular rectangle or trapezium, the engineer must proceed to determine the figure of the transverse area without violating the conditions they impose.

115. When it is desired to convey the greatest possible quantity of water in an open channel with a given area of transverse section, then the volume discharged being directly proportional to the area, and inversely as the wetted border, we must select the figure which for a given area has the least border, and for a given border has the greatest area.

Geometry informs us that the circle has this property: the semicircle, and therefore the semicircular channel, has the same property; the ratio between the area of the semicircle and semi-circumference being the same as that between the circle and the entire circumference. Then follow the regular demi-polygons, with less and less advantage as the number of their sides is less; and among the more practicable forms are the demi-hexagon, and finally the half-square.

As the transverse areas of artificial open channels are, when without masonry, trapezoidal, the question as to the figure of greatest discharge is reduced to taking, among all the trapeziums with sides of a determinate slope, that which gives the greatest section for a given wetted border; or, in other words, which has the greatest hydraulic mean depth; and every different area and ratio of slopes has its particular maximum trapezium.

Let p be the depth of the trapezium BF (Fig. 50), and b the bottom width BC, and $n : 1$ the ratio of the slopes, or AF : FB; then the general values of S and C are

$$(p) \quad . \quad . \quad . \quad . \quad S = (b + np) \times p = bp + np^2,$$

and

$$(q) \quad . \quad . \quad . \quad . \quad . \quad C = b + 2p\sqrt{n^2 + 1}.$$

Since, then, S in the expression $\frac{S}{C}$, with slopes of $n:1$, is a maximum, its differential will be zero, and we have

$$(r) \quad . \quad . \quad . \quad . \quad . \quad pdb + bdp + 2npdp = 0\,;$$

and as the border is constant, its general value being differentiated, gives $db + 2dp\sqrt{n^2+1} = 0$. Hence $db = -2dp\sqrt{n^2+1}$; this being substituted in (r), gives $b = 2p\,(\sqrt{n^2+1} - n)$; with which value of b we have

$$(s) \quad . \quad . \quad . \quad . \quad \frac{S}{C} = \frac{p^2\,(2\sqrt{n^2+1} - n)}{2p(2\sqrt{n^2+1} - n)} = \frac{p}{2}.$$

Therefore, in all trapezoidal channels of the best form, with certain given slopes and area, the hydraulic mean depth is half the depth of the water: and hence we derive a construction for the cross section of a maximum discharging channel; remarking that as $\frac{S}{C} = \frac{p}{2}$, we have $S = C \times \frac{p}{2}$. Let the trapezium, ABCD (Fig. 51) be the channel sought; from the middle point E of the top width draw lines EB and EC dividing the figure into three triangles, of which AEB and CED are identical; let EP be the perpendicular from E upon AB; then

$$\overline{AB+BC+CD} \times \frac{p}{2} = \overline{AB+CD} \times \frac{EP}{2} + BC \times \frac{p}{2}\,;$$

and therefore $\frac{p}{2} = \frac{EP}{2}$. If, then, from E as centre, and p as radius, we describe a circle, it will touch the two sides AB and CD. If, therefore, we describe a circle (Fig. 52) with any radius, and draw a tangent, parallel to a horizontal diameter, produced on each side indefinitely, then between these lines drawing tangents having the given inclination, we obtain a figure similar to that required, from which, by proportion, we obtain the transverse section of the channel: a construction given by Mr. Neville (Hydraulic Tables, p. 129).

Other properties of the trapezium of greatest discharge, Figs. 51 and 52, are, First, that the line of surface of water AD is equal to the sum of the slopes AB and CD, and consequently the border is equal to the sum of the top and bottom widths, or the mean breadth equal half the border. Secondly, the triangle BEC is similar to the triangles EAB and EDC; the vertical angle BEC being equal to the angle of inclination of the sides to the horizon. Thirdly, the angle between the perpendiculars from E upon the sides AB and CD is double the angle of inclination of the sides, and the angle PEB half of the same, that is, of the angle BEC. This gives another construction when p and the angle of inclination of sides are given. On a vertical line lay off p, and from the upper point E (Fig. 51), on each side, lay off the angle of inclination, bisect them by EB and EC, and through the lower point of p draw a perpendicular to intersect EB and EC, which gives the base BC, then from B draw BA perpendicular to EP, to intersect the horizontal line through E at A, and in like manner on the opposite side, giving the required trapezium ABCD. From the second we obtain an expression for the area of the trapezium of greatest discharge in terms of the depth and angle of inclination β of the side slopes with the horizon, for the area of the triangle BEC is equal to $p \times p \tan \frac{1}{2}\beta$, as half $\overline{BC}$ is the tangent of half the vertical angle to radius p; also the sum of the areas of the triangles EAB and EDC is equal to $\overline{EP} \times \overline{AB}$, but EP is equal to p, and $\overline{AB}$ is the cosecant of β to radius p, as is evident if from B we draw the perpendicular BF, as in Fig. 50, the angle ABF being the complement of BAF, that is β; thus the area of the trapezium is

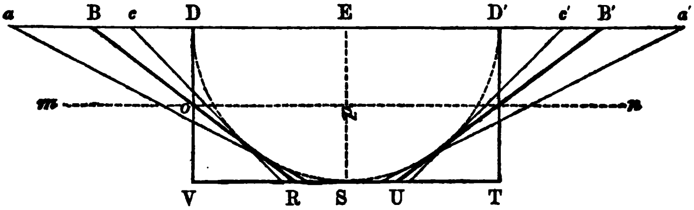
$$p^2 (\tan \tfrac{1}{2}\beta + \operatorname{cosec} \beta).$$

TABLE *giving the values of* $\tan \frac{1}{2}\beta + \operatorname{cosec} \beta$, *and the Top and Bottom Widths, in Trapeziums of best Form and ordinary Slopes.*

Slopes.	Angle β.	$\tan \frac{1}{2}\beta$ $+ \operatorname{cosec} \beta$	Top width.	Bottom width.
0 to 1	90°	2.000	$p \times 2.000$	$p \times 2.000$
¼ ”	75 58′	1.812	2.062	1.562
½ ”	63 26	1.736	2.236	1.236
⅗ ”	60	1.732	2.309	1.155
¾ ”	53 8	1.750	2.500	1.000
1 to 1	45	1.828	2.828	0.828
1¼ ”	38 39	1.952	3.202	0.702
1⅓ ”	36 53	2.000	3.333	0.666
1½ ”	33 41	2.106	3.606	0.606
1¾ ”	29 44	2.282	4.032	0.532
2 to 1	26 34	2.472	4.472	0.472
2¼ ”	23 58	2.674	4.924	0.424
2½ ”	21 48	2.885	5.385	0.385
2¾ ”	19 58	3.104	5.854	0 354
3 to 1	18 26	3.325	6.325	0.325

The several numbers in the third column express the areas of the trapeziums of best form to the depth unity, and also the mean widths: multiplied by p^2 they give the area, and by p the mean width for the depth p. The numbers under the fourth and fifth columns are the top and bottom widths to the depth unity, and for any depth p give the top and bottom width respectively, by being multiplied into p; they are obtained by adding, for the fourth column, the numbers in the first and third, and for the fifth column, by subtracting the same, for in every trapezium the top width is equal to the mean width, plus the depth into the ratio of the slopes, and the bottom width equal to the mean, minus the depth into the ratio of the side slopes, and with a depth unity the value of these lines is derived by the addition and subtraction above mentioned.

In all the different trapeziums of best discharge formed, as shown in the woodcut, by drawing tangents to the same semicircle having the radius p, the hydraulic mean depths are evidently the same, namely, $\frac{1}{2}p$, whatever the side slopes may be, and, therefore, with the same inclination of the bed of the channel, they all have the same velocity, and consequently the discharge, which will then be as the area, is proportional to the mean width, the depth p being constant.



If through the middle point of p we draw the indefinite line mn parallel to aa', then the areas of all these figures formed by the several tangents to the semicircle will be proportional to the length of this line cut off by the tangents or side slopes; this consideration serves to explain what may be observed in the last Table, namely, that the numbers in the third column decrease from the first number, corresponding to 0 to 1, down to a certain point, and then, rising to the first value at the slope of $1\frac{1}{3}$ to 1, afterwards increase continuously. In the woodcut the vertical tangents at D and D′ form the rectangle of best discharge with a given area, and the length of mn cut off is equal to twice the depth p; now every other tangent between that from D and that from B, which passes through the middle point O of DV (and, therefore, forms a trapezium BRUB′, having the same mean width as the rectangle), cuts off a smaller part of the line *mn*, and consequently is of a less area than the rectangle, the minimum being the trapezium formed by the tangent which touches the semicircle at the point in which the line *mn* cuts it, and having therefore the inclination of 60° with the horizon, the mean width, to the depth unity, being 1.732, and the trapezium a half regular hexagon; but the tangents cutting *mn* beyond the point O form trapeziums of a continually increasing mean width, and, therefore, increase in area in the same proportion. If through the point O we draw any inclined lines whatever, the areas of the figures so formed are all equal to that of the rectangle, but that particular line which, drawn through O, also touches the semicircle, forms a trapezium, whose wetted border, as well as area, is equal to that of the rectangle, for the slope BR is equal to the half top width BE, which is equal to the radius DE (that is p or DV), together with BD, which is equal to VR, and, therefore, BR is equal to DV + VR, and consequently $\overline{BRUB'} = \overline{DVTD'}$.

Also the slope of BR is $1\frac{1}{3}$ to 1, or, which is the same thing, VR = $1\frac{1}{3}$ of OV; for the sum of the sides of the triangle OVR is evidently equal to DV + VS, or 2DV, or 4OV; take OV from both, and we have

$$3OV = OR + RV, \text{ and also } OR^2 - RV^2 = OV^2,$$

that is, $$\overline{OR + RV} \times \overline{OR - RV} = OV^2;$$

substituting $$3OV \times \overline{OR - RV} = OV^2,$$

hence $$OR - RV = \tfrac{1}{3}OV;$$

subtracting this last from

$$OR + RV = 3OV,$$

we have $$2RV = 3OV - \tfrac{1}{3}OV,$$

and, therefore, $$RV = 1\tfrac{1}{3}OV.$$

The three sides of the triangle are consequently as the numbers 5, 4, 3: and in this trapezium, the bottom width is $\frac{1}{3}$th of the top, but this last relation as to the widths is not needed to the equality of both area and border in a trapezium and rectangle, for if the slopes of the former be $1\frac{1}{3}$ to 1, and the vertical sides of the rectangle bisect them, then, however great may be the bottom width, or even if it disappear and the slopes meet in a point, the condition holds, and the rectangle and trapezium have the same discharge, velocity, and hydraulic mean depth consequent on their identity of border and area.

In the woodcut, p. 139, the tangents from *a* and *a′* are at a slope of 2 to 1; those from *c* and *c′* are at 1 to 1. It is remarkable how small relatively the bottom width becomes as the slope of the sides becomes flatter, at 1⅓ to 1, being but two-thirds of the depth and one-fifth of the top width; at 3 to 1, being one-third of the depth and a tenth of the top width; for flatter slopes than this last, the trapezium of best form may be considered practically to merge into a triangle. If the top width with the flatter slopes be considered to involve too great an expenditure in land, and that the upper part of the excavated ground be of a nature to bear a steeper slope than the lower, then tangents to the semicircle with that slope will give a figure of best discharging form for a given area, and with those conditions, economizing both land and excavation, instances of such as having been adopted in practice are given under the Examples. The bottom, also, of the channel may be constructed curvilinear by adopting for it the arc of the semicircle between the points at which the side slopes touch it, and which arc, therefore, subtends an angle at the centre equal to twice the angle of inclination of the sides. In these two last modifications the hydraulic mean depth, and therefore the velocity, are evidently the same as in the simple trapezium, and the discharge diminished only as the area of either is diminished by the omitted portions of the original trapezium.

In order to compare a trapezoidal channel of best discharging figure with others having, first, the same constant border; secondly, the same constant area, and in both the inclination of the side slopes identical, we may proceed as in pp. 129, 130, in which were tabulated the results upon the hydraulic mean depth and discharge of all the different figures which a transverse section of a rectangular form may assume when the border is constant, and again when the area is constant.

Let us suppose a trapezium with side slopes of 30° with the horizon, that is, 1.732 to 1 (nearly 1¾ to 1), and a constant border of 200 units. It is evident (Fig. 73) that when we diminish the depth, the bottom width increases, and that the limiting figure is with a depth equal to zero, a bottom width equal to 200. On the other hand, when the bottom width is equal to zero, and the depth the greatest possible, the trapezium becomes an isosceles triangle, whose base angles are equal to the angle of inclination of the sides, and consequently unlike the former limit (and both limiting figures in the rectangle of constant border) having a finite area.

TABLE *showing the value of the Bottom Width, Border, and Hydraulic Mean Depth, &c., the transverse section being a trapezium with side slopes* 30° *with horizon, and a constant border equal* 200 *units.*

Depth in feet.	Bottom width.	Area.	Hydraulic mean depth.	Square root of hydraulic mean depth.
0	200	0	0	0
1	196	197.7	0.99	0.99
2	192	390.9	1.96	1.39
3	188	579.6	2.90	1.70
5	180	943.3	4.72	2.17
10	160	1773.2	8.87	2.97
20	120	3092.8	15.46	3.93
25	100	3582.5	17.91	4.23
30	80	3958.8	19.79	4.45
35	60	4221.8	21.11	4.58
40	40	4371.3	21.86	4.67
41	36	4387.6	21.94	4.68
42	32	4399.3	22.00	4.69
43	28	4406.6	22.03	4.694
44	24	4409.25	22.04	4.695
44.09	23.63	4409.27	22.05	4.696
45	20	4407.3	22.04	4.694
50	0	4330.1	21.65	4.653

Taking in the next place a constant transverse area for the trapezium, as in Fig. 74, whose side slopes are at 30° with the horizon, and supposing this area to be that which was the maximum in the above Table, 4409.27 square units: we find the mean widths by dividing this number by the several depths we assume; the quotient is the mean width of the trapezium, from which the top and bottom widths are deduced by adding and subtracting the product of the depth into the ratio of the slopes.

From this it readily appears that there is a limit to the depth, for, when the product above mentioned is equal to the mean width, the difference is zero, and the figure becomes an isosceles triangle, whose base angles are equal the angle of inclination of the sides, and area equal the constant area chosen. But the bottom width increases without limit as the depth chosen diminishes, and at a depth equal to zero becomes infinite—

TABLE *showing the value of the Bottom Width, Border, and Hydraulic Mean Depth, &c., the transverse section being a trapezium, with side slopes* 30° *with horizon, and the constant area* 4409.27 *square units.*

Depth.	Bottom width.	Border.	Hydraulic mean depth.	Square root of hydraulic mean depth.
0	∞	∞	0	0
1	4407.5	4411.5	0.99	0.99
2	2201.2	2209.2	1.99	1.40
3	1464.6	1476.6	2.98	1.73
5	873.2	893.2	4.93	2.22
10	423.6	463.6	9.50	3.08
20	185.8	265.8	16.60	4.07
25	133.1	233.1	18.92	4.35
30	95.0	215.0	20.50	4.53
35	65.3	205.3	21.46	4.63
40	40.9	200.9	21.95	4.685
41	36.5	200.5	21.99	4.689
42	32.2	200.2	22.02	4.692
43	28.1	200.1	22.03	4.694
44	24.0	200.0	22.04	4.695
44.09	23.6	200.0	22.05	4.696
45	20.0	200.0	22.04	4.695
50	1.6	201.6	21.88	4.67
50.455	0	201.8	21.85	4.674

In both the Tables it is remarkable how nearly equal the numbers in the fifth column are, on each side of the maximum, even for a wide range of values in the depth and bottom width, and this has an important practical bearing, for if, page 142, a depth of 44 and bottom width of 24 units were found inapplicable or expensive from the great depth the excavated earth would have to be raised, we may adopt a channel having a depth of 30 and bottom width of 95 units, and as the transverse area is constant, the volume discharged will be influenced only by the alteration in the mean velocity, that is, in the value of the square root of the hydraulic mean depth, and this we perceive is only reduced from 4.695 to 4.53, which is less than 3½ per cent.; the longitudinal inclination of the bed of the channel being the same in both cases. The land required for the wider channel would be about 12 per cent. greater than for that of best discharging form. In forming a new or an improved river channel, the excavated earth is almost always carried to spoil on each side, and not to a contiguous embankment, as in road or railway works, which makes the *depth* of the cutting of the greater importance with a view to economy.

In all cases the top width spoken of is supposed to be the level of high water of the greatest floods, and should be 3 or 4 feet below the surface of the land on each side, in order that the thorough drainage may not be injuriously affected.

116. The mean velocity of water flowing in an open channel is about 4-5ths of the maximum velocity, which is generally at the centre and upon the surface, or a little below it; and, conversely, the maximum velocity at the surface is found from the mean velocity by adding a fourth (Minard, "Cours de Construction," p. 6).

Thus, if the mean velocity be 3 ft. per sec., that at the surface is $\frac{5}{4} \times 3 = 3.75$ ft. per sec.

Also, if the observed central velocity at the surface were found to be 5.2 ft., the mean velocity is 4.16 ft. per sec.

Dubuat has given an empiric formula, on which have been founded Tables by most of the authors of Hydraulic Tables.

In the formulæ for the velocity and discharge of open channels and pipes given in this Chapter, the direction of both the pipe and channel is supposed to be nearly a right line: when they have quick curves, an additional resistance is occasioned, which diminishes the discharge, or demands an increased head to give any required discharge. This resistance is said to depend conjointly upon the square of the velocity of the water, upon the number of bends, and on the square of the sine of angle they make with the straight line of direction; and Mr. Beardmore has added, inversely, as the square root of the hydraulic mean depth; but experimenters have not been consistent in the results obtained (D'Aubuisson, §§ 196–198).

117. In the case of pipes running full, the bends may occur in the vertical plane also; and in this case the air is found to collect rapidly at the summit of such bends: air-valves must, therefore, be left to free the pipe, which may be in some cases self-acting, but are generally worked by hand at stated times.

It is also necessary to proportion the diameter of the pipe in the different parts of its course, so as to make it discharge the quantity due to its diameter. Thus, in Fig. 53, which is the longitudinal section of the line of a main pipe leading the water from a reservoir at A to the point Y, if from the formation of the intervening ground it is necessary to rise up to the point B, then it is impossible for the pipe to discharge at its extremity, Y, the amount due to the diameter and *total fall* AM, from A to Y; neither can the fall from B to Y be fully efficient, because there cannot be a due supply at B. The pipe from A to B must therefore be of greater diameter than from B to Y, or than would have been necessary if the total fall from A to Y had been uniformly distributed. At the enlargement of the Edinbugh Waterworks, as designed by Mr. Jardine, the main for the first 18,300 ft. had a fall of 65 ft.; and the diameter, commencing at 20 inches, decreased to 18 inches: the remainder of the distance, 27,900 ft., had a fall of 286 ft., and a diameter of only 15 inches. The discharge into the Castle Hill distributing reservoir is only that due to the smaller diameter, laid the whole distance with an uniform fall.

118. On this main were placed, at fourteen different points, —the summits of bends in the vertical plane,—cast-iron vessels to receive the compressed air as it collected. Fig. 54 shows a vertical section of one of them, 4 ft. high, and 1.5 ft. wide, with the cock for letting off the air, which must be done every three or four days. The neglect of this precaution has been the cause of great disappointment upon the first opening of waterworks.

EXAMPLES AND PRACTICAL APPLICATIONS.

CHAPTER III.

119. (XLIV.) It frequently occurs that new channels have to be excavated in soils having a degree of cohesion such as will only bear a certain limit of velocity; if this be exceeded, the water carries down the particles of the bottom and sides of the bed, destroying its form, and also injuring those parts of the course lower down, where from a lesser inclination of the downward slope, and consequently lesser velocity, it is deposited, an evil of an exactly opposite character to the first.

The volume of water brought down by the stream in its various states, more especially in floods, being known (*vide* Ex. XXX.), and also this limiting velocity; it is required to design a suitable form of channel. If it is intended to be a trapezium, and without any masonry or stone pitching on the side slopes, then a second condition, depending in like manner on the nature of the soil, is introduced, namely, the inclination of the slide slopes, which must not be so steep as to endanger their slipping in, this latter condition being common to all excavations, for whatever object they may be undertaken. Now the volume and mean velocity being known, we have at once the transverse area of the channel in what form soever it may be distributed. Calculate the area of transverse section of a channel required to convey 104 cb. ft. per sec., at a velocity of 2.5 ft. per sec. Solving the expression $Q = Sv$ for S, we have

$$S = \frac{Q}{v}; \text{ hence, } \frac{104}{2.5} = 41.6 \text{ sq. ft.}$$

for the area, S; and if together with the ratio of the side slopes, $n : 1$, we are given either the depth p, or the top or bottom width, the trapezium is completely determined.

For, if the depth be given, we obtain the mean width by dividing the area by the depth; and from the mean width, the top or bottom width is found by adding or subtracting $p \times n$.

Thus, if the depth $p = 3$ ft. and $n = 2$, or the slopes 2 to 1, the bottom width b is $\frac{41.6}{3} - (2 \times 3) = 7.866'$ ft.; and the top width, in like manner, is $\frac{41.6}{3} + (2 \times 3) = 19.866'$ ft. If the bottom width b be given to find p, we have $bp + np^2 = 41.6$ sq. ft. Let $b = 8$, hence, $p = \pm\sqrt{24.8} - 2 = 2.98$ ft.

It must be observed, that the inclination of the bed on the longitudinal section of the channel is not, with the above data, a matter of choice, but is absolutely determined by the conditions as to volume, velocity, and dimensions of transverse section.

In the former, when $p = 3$ ft., we have $\frac{S^3}{C} = \frac{41.6^3}{21.43} = 3359.4$; and from equation ($o$) the inclination $\frac{h}{l} = \left(\frac{104}{100}\right)^2 \times \frac{1}{3359.4} = 0.000322$, or 1 in 3106, being 1.7 ft. per mile; and in the latter, $\frac{h}{l} = 0.0003204 = \frac{1}{3122}$.

To determine the dimensions of a rectangular channel formed of timber, or with vertical sides of masonry, to convey the same volume of water at the same velocity. The best form of rectangular channel being that in which the depth is half the breadth, we have the depth found from the expression—

$$p = \sqrt{\frac{Q}{2v}} = \sqrt{20.8} = 4.56 \text{ ft.}$$

and consequently the width 9.12 ft.

Calculate the dimensions of a regular half hexagon to convey the same volume of water with the same velocity. We have the area of regular hexagon equal to $s^2 \times 2.598$, s being one of the sides. Hence—

$$s^2 \times 1.299 = 41.6, \text{ or } s = \sqrt{\frac{41.6}{1.299}} = 5.66,$$

and hence the depth $p = 4.9$ ft., and $C = 16.98$ ft.

If the trapezium was required to be of the best discharging form with side slopes, as above, 2 to 1, then from Table, page 138, we have the area equal $p^2 \times 2.472$, and therefore the equation

$$p^2 \times 2.472 = 41.6 \text{ sq. ft.},$$

and

$$p = \pm\sqrt{\frac{41.6}{2.472}} = 4.102.$$

Proceeding as above, when $p = 3$ ft., we now have the bottom and top width respectively equal to 1.94 and 18.345 ft.

(XLV.) Having a pipe of 2 ft. diameter, and 4000 ft. in length, with a head of 80 ft., calculate the discharge per minute. We have by equation (h)—

$$2356\frac{\sqrt{2^5}}{\sqrt{\frac{400}{80}}}=2356\frac{5.657}{7.071}, \text{ or } 2356\times 0.8 = 1884.8 \text{ cb. ft. per min.}$$

(XLVI.) Calculate the respective discharges by pipes 1 ft. and 4 ft. diameter, the length and head being the same as in the last example.

For the pipe 1 ft. in diameter:—

$$2356\frac{1}{\sqrt{\frac{4000}{80}}}=\frac{2356}{7.071}=333.21 \text{ cb. ft. per minute.}$$

And for that 4 ft. in diameter:—

$$2356\frac{\sqrt{4^5}}{7.071}=2356\frac{32}{7.071}=2356\times 4.5255=10{,}662 \text{ cb. ft. per min.}$$

Thus the transverse areas in the three cases being as 1 : 4 : 16, that is as d^2, the discharges are as 1 : 5.656 : 32, or as $d^{2.5}$; all the other data being the same.

(XLVII.) Calculate the discharge of a pipe whose area is one-half that of the 2-ft. pipe in (XLV.), with the same length and fall. The areas being as 2 : 1, the diameters are as 1.414 : 1, the $\sqrt{2}$ being = 1.414. Hence—

$$2356\frac{\sqrt{1.414^5}}{7.071}=792 \text{ cb. ft.};$$

and two such pipes discharge 1584 cb. ft. per minute, being 300 cb. ft. per minute less than the single 2-ft. main of equal area.

(XLVIII.) Calculate the diameter when two equal pipes give the same discharge as that of the single pipe in (XLV.) This is the same as to calculate the diameter of a pipe which shall discharge 942.4 cb. ft. per minute, with a length of 4000 ft., and head of 80 ft. Hence, if x be the diameter sought, we have—

$$2356\times\frac{\sqrt{x^5}}{\sqrt{\frac{l}{h}}}=942.4 \text{ cb. ft.}$$

And—

$$\left(\frac{942.4}{2556}\times\sqrt{50}\right)^2=x^5;$$

or—

$$x = \sqrt[5]{(0.4 \times 7.071)^2} = \sqrt[5]{8} \text{ and } \frac{\log 8}{5} = \frac{0.90309}{5} = 0.180618,$$

answering to the number 1.5157; being 0.1017 ft., nearly $1\frac{1}{4}$ in., greater than 1.414 ft., and discharging 150 cb. ft. more.

(XLIX.) Calculate the diameter of one large main, to convey the same quantity of water as three mains, each 2.5 ft. in diameter; the length $2\frac{1}{2}$ miles; head, 140 ft. The quantity that may be delivered by the three mains with the above data is—

$$3 \times 2356 \frac{\sqrt{2.5^5}}{\sqrt{\frac{13200}{140}}} = 7200.7 \text{ cb. ft. per minute.}$$

for log 2.5 = 0.39794 and $\frac{5 \times 0.39794}{2} = 0.99485$, answering to the number 9.8821; also as $\sqrt{\frac{13200}{140}} = 9.7$, and $\frac{9.8821}{9.7} = 1.0188$: so that $3 \times 2356 \times 1.0188 = 7200.69$ cb. ft. per minute, as above.

Hence, the diameter of the single main being d, we have from (k)—

$$d = \sqrt[5]{\left(\frac{7200.7}{2356}\right)^2 \times 94.3} = \sqrt[5]{3.0563^2 \times 94.3},$$

or by logs $\frac{2 \times 0.485196 + 1.9745117}{5} = 0.5881874$, answering to 3.8742 ft., about 3 ft. $10\frac{1}{2}$ in.; so that an addition of 18 in. to the diameter of one of the 30 in. mains, gives a discharge equal to all three of 30 in. diameter. The last five questions point out very clearly the great economy of adopting at the first a large diameter of pipe: the thickness of the cast-iron of the pipes being in practice very little, if any, diminished in the lesser diameters, and the circumference being as d, the discharge is as $d^{2.5}$ *cæteris paribus*. In this example the quantity of cast-iron in the one large pipe is to that in the three smaller as 3.8915 to 7.5.

(L.) We may, from a consideration of the formula—

$$(j) \quad . \quad . \quad . \quad . \quad . \quad 0.9166 \sqrt{2fH_y} = v,$$

the mean velocity, explain several of the circumstances of rivers and channels. Thus, in time of floods, it is observed that the velocity is largely increased, although the slope of inclination of the surface, if it be not the same as in the ordinary and average state, is probably even smaller in some cases. Thus, let us suppose $a' a n b b'$ (Fig. 55) to represent the transverse section of a river, at some point where it is of a regular figure, with the same slopes on each side; let a, b be the level of the ordinary water, and a', b' that of a high flood: the transverse area S increases faster than the wetted border C; and therefore $\frac{S}{C}$, the hydraulic mean depth, increases faster than C; consequently the mean velocity also increases with the depth of the water, and in proportion to the square root of the hydraulic mean depth.

Let us suppose a river 68 ft. broad at the bottom, and the slopes 2 to 1; the ordinary depth 4 ft., and the fall per mile 1.4 ft.; we have therefore—

$$v = 0.92\sqrt{2 \times 1.4 \times \frac{304}{85.88}} = 2.7 \text{ ft. per sec.,}$$

and the discharge is $2.9 \times 304 = 881.6$ cb. ft. per sec.

If, then, in time of flood, the river rises so as to have a depth of 7 ft., the fall per mile being supposed unaltered, we have now, consequently, the mean velocity expressed in numbers—

$$v = 0.92\sqrt{2 \times 1.4 \times \frac{574}{99.304}} = 3.7 \text{ ft. per sec.}$$

And the discharge $3.7 \times 574 = 2123.8$ cb. ft.; being 2½ times greater than the former, although the transverse section is only 1.888 times, and the depth but 1.75 times greater.

(LI.) It has been observed at the junction of rivers of nearly equal width and volume, such as the Mississippi and Ohio, the Ganges and Jumna, &c., that the width is not perceptibly increased below the junction: the velocity and depth are increased, so that the double volume of water is conveyed in a channel of nearly the same width; the inclination of the surface being also the same before and after the junction. Required the new depth and mean velocity of the united streams.

Let d be the depth of each of the equal streams, and b the mean width, we may assume the border to be $b + 2d$, and the hydraulic mean depth $\frac{bd}{b + 2d}$.

Thus, each river, before the junction, discharged—

$$0.92\sqrt{2f\frac{bd}{b + 2d}} \times bd,$$

and if we call x the depth of the single river after the union of the streams, and breadth b, as before, we shall have—

$$Q = 0.92\sqrt{2f \times \frac{bx}{b+2x}} \times bx = 2 \times 0.92 \times \sqrt{2f \times \frac{bd}{b+2d}} \times bd.$$

Hence—

$$\sqrt{\frac{bx}{b+2x}} \times bx = 2\sqrt{\frac{bd}{b+2d}} \times bd.$$

Squaring both sides, and dividing out by b^3, we have

$$\frac{x^3}{b+2x} = 4\frac{d^3}{b+2d}.$$

Clearing of fractions and transposing—

$$x^3 - \frac{8d^3}{b+2d}x = \frac{4bd^3}{b+2d}.$$

Let $b = 200$ ft., and $d = 5$, then—

$$x^3 - 4.762\,x - 476.2 = 0.$$

After a few substitutions, we obtain $x = 8.01$ ft. If $f = 3$, the mean velocity in the rivers, before the junction, is 4.92 ft., and of the single united stream 6.14 ft. per second.

(LII.) Calculate the quantity of water conveyed by a channel 4 ft. deep, 18 ft. wide at top, and 7 ft. at bottom, and the inclination of the surface 4 in. per mile.

The length of each of the side slopes is 6.8 ft.; hence the border is 20.6 ft., and the area 50 sq. ft., from which we have the hydraulic mean depth—

$$\frac{S}{C} = \frac{50}{20.6} = 2.4272 \text{ ft.}$$

Hence from (j)—

$$v = 0.92\sqrt{2 \times 0.333' \times 2.4272} = 1.17 \text{ ft. per sec.,}$$

and $Q = 50 \times 1.17 = 58.5$ cb. ft. per sec.

Let the slope of the above channel be doubled; required now the mean velocity and discharge, the depth and other dimensions of the transverse section remaining the same.

It is evident that the only change required in the value of the expression for v—as given in LII.—is to multiply by

$\sqrt{2}$; for as the hydraulic mean depth is 2.4272, as before, we have $v = 0.92 \sqrt{2 \times 0.666' \times 2.427} = 1.654$ ft. per sec., and $Q = 50 \times 1.654 = 82.7$ cb. ft. per sec.; and, generally, the discharges are, *ceteris paribus*, as the square roots of the inclinations; so that if the fall had been increased four times, the discharge would have been doubled.

(LIII.) In the mining districts of Cardiganshire, it is usual to collect the supplies of water for the stamping-mills, &c., by channels, called leets, often many miles in length, and frequently deriving the water from sources altogether out of the limit of the natural watershed of the point where the power of this water is applied. The fall given to these channels is generally 1 inch in 10 fathoms, or 1 in 720, being 7.33′ ft. per mile.

Calculate the quantity of water brought down by a leet having this fall: depth, 0.5 ft., and bottom width, 2 ft.; slopes, $1\frac{1}{2}$ to 1.

The area is 1.375 sq. ft., and the border

$$2 + 2 \times 0.5 \sqrt{1.5^2 + 1} = 3.80 \text{ ft.}$$

Hence—

$$\frac{S}{C} = 0.362,$$

therefore the velocity is $= 0.92 \sqrt{2 \times 7.33' \times 0.362} = 2.119$ ft. per sec., and $Q = 1.375 \times 2.119 = 2.9$ cb. ft.

These streams have sometimes to be carried along the face of rocks, nearly perpendicular, on the side of a valley. In such places the water is conveyed in wooden troughs, supported by iron holdfasts. Calculate the transverse section of a trough to convey 2.7 cb. ft. at the above velocity. The wooden trough is rectangular, the depth half the breadth, hence the side of the square of which its transverse area is the half is—

$$= \sqrt{\frac{2Q}{v}} = \sqrt{\frac{2 \times 2.9}{2.119}} = 1.654;$$

the depth, then, is 0.83 ft., and width, 1.654 ft.

(LIV.) Calculate the quantity of water conveyed by a channel of the following dimensions:—Bottom width, 10 ft.; slopes, 2 to 1; depth, 4 ft.; and fall per mile, 0.5 ft.

$$S = p^2n + pb = 4^2 \times 2 + 10 \times 4 = 72 \text{ sq. ft.},$$

$$C = b + 2p \sqrt{n^2 + 1} = 10 + 2 \times 4 \sqrt{5} = 27.9 \text{ ft.},$$

and

$$\frac{S}{C} = 2.58,$$

hence,

$$Q = 72 \times 0.92 \sqrt{2 \times 0.5 \times 2.58} = 106.38 \text{ cb. ft.}$$

If the fall had been 3 ft. per mile, then the value of Q, above found for 0.5 ft. fall, must be multiplied by $\sqrt{6} = 2.449$, and the discharge would be 260.5 cb. ft. per sec.

Let the fall per mile be reduced from 0.5 ft. to 0.125 ft. per mile, i. e. the fourth part, then the volume of water discharged will be reduced one-half, being equal to 53.19 cb. ft. per second.

Or, if the fall remain unaltered, at 0.5 ft. per mile, and the other factor under the radical sign, that is, the hydraulic mean depth, be reduced in the same proportion as the fall had been above, namely, from 2.58 ft. to 0.645 ft. (the transverse area of 72 sq. ft. remaining constant), the discharge would in this case also have been 53.19 cb. ft.

To construct this trapezium we have the area 72, and the hydraulic mean depth equal 0.645, which gives the border 111.6, hence by a quadratic we find the depth 0.655 ft.; rejecting the other solution when the product of the depth into the cosec. of inclination of sides is greater than half the border: and the bottom width 108.67 ft., such a transverse section, from its great relative resistance, has the same effect in diminishing the discharge as the lowering of the fall from 0.5 ft. to 0.125 ft. per mile.

If we desire to have a channel of best form, 72 sq. ft. area, and slopes 2 to 1, we have $p^2 \times 2.472 = 72$, and hence p the depth equal 5.4 ft., the hydraulic mean depth 2.7 ft., the bottom width 2.5488 ft., border 26.73 ft., and the discharge 108.6 cb. ft.

(LV.) Calculate the quantity of water flowing out of a railway tunnel by the two side drains, which were built with vertical walls of masonry; each channel being 1 ft. wide, and the water 5 inches deep; the inclination of the tunnel being 1 in 200.

$$Q = 2 \times \left\{0.4166' \text{ sq. ft.} \times 0.92 \sqrt{2 \times 26.4 \times \frac{0.4166'}{1.833'}}\right\} = 2.65 \text{ cb. ft.}$$

As the depth is 0.4166′ ft., and breadth, 1 ft., the value of S is evidently 0.4166 sq. ft., and the border, $1 + 2 \times 0.4166 = 1.8333'$. The fall per mile for 1 in 200 is 26.4 ft.

120. The actual rate of fall adopted by engineers may be

studied in the following series of examples, from which we may gather the fact that in artificial channels, as in rivers the work of nature, large volumes of water must have but small inclinations of their bed; otherwise the velocity resulting from the smaller relative resistance would overcome the cohesion of the bed and banks.

The celebrated Shawswater aqueduct or channel, which conveys water to Greenock for mill-power and supply of the town, has a fall of 4 ft. per mile throughout its length of 5 miles; it discharges 43.3 cb. ft. per sec., equal to 2600 cb. ft. per minute, the depth of water being about 1.56 ft.; bottom width, 6 ft., and side slopes generally, ½ to 1.

The canal from the River Durance to Marseilles (LIX.), intended for irrigation, water-power, and supply of the city, has an average fall of 2.416 ft. per mile, conveying 528 cb. ft. per sec.

The Grand Ganges Canal (LVII.), now opened, the largest in the world, having a course of 898 miles, navigable throughout, and furnishing irrigation to 5,400,000 acres, has a fall of 1.5 ft. per mile, a depth of 10 ft., with a constant bottom width of 150 ft.; and is intended to convey 6750 ft. per sec.; the slopes are not mentioned, but at 1½ to 1 the formula would give 7316 cb. ft. The fall in this canal may, from its vast dimensions of transverse section, be considered very great, and the resulting velocity such as would ordinarily wear down the sides—in tropical countries, however, it would appear that the growth of aquatic plants, causing a great increase of resistance on the bottom sides, is a serious evil to canal works, and necessitates a greater inclination of surface than would be required in higher latitudes.—("Italian Irrigation," by Captain B. Smith.)

Covered channels constructed of masonry offer less resistance from the greater truth of the lines of the work, and evenness of surface of bottom and sides of channel, when well constructed, than is found in open channels excavated in the ground.

The Croton Waterworks, supplying the city of New York, afford the finest modern example of water supply having this great advantage of a covered channel, so essential to the purity and low temperature of the water. The transverse section of the culvert is given, with figured dimensions, in Fig. 55. The fall is 13½ inches, or 1.125 ft. per mile; the discharge is said to be 60,000,000 gallons in twenty-four hours, which gives 111.1 cb. ft. per sec.

The supply of water from Loch Katrine for the city of Glasgow is conveyed by a culvert or tunnel 8 ft. in diameter, and with a fall of 1 in 6336; in crossing the intervening valleys, a cast-iron pipe, 4 ft. in diameter, is used, like an inverted syphon; the total fall from point of entry to that of delivery being at the rate of 1 in 1000, or 5.28 ft. per mile.

The Canal de l'Ourcq, conveying water from the Ourcq to Paris, has an inclination of 0.0001056, or 33.26 ft. fall in the total length of 314,966 ft. The gauging of the river l'Ourcq showed that 106.61 cb. ft. were to be conveyed down; the projected navigation required a depth of 4.9213 ft., and it was deemed necessary to give a velocity to the current of 1.1483 ft. per sec. From these data it resulted, from the use of the complicated formula which introduced the simple power of the velocity (p. 108), and such other modifications of it as were assumed to represent every different kind of resistance, that the inclination should be 0.00005502. But the engineer, fearing the retarding effect of the weeds and plants which grow abundantly on the bottom and sides, finally adopted that named above, 0.0001056, or nearly double that given by calculation, making the fall 0.5575 ft. per mile.

The New River (as it is still called), conveying water to London from near Ware on the River Lea, is graduated to 0.25 ft. per mile, having a winding course of 39 miles; its breadth is said to be 18 ft., and depth, 4 ft. The fall in this canal, intended for domestic supply, is too small, as in summe the water becomes raised in temperature from the great surface exposed in its long course, and from the slow rate of motion, which is about half a mile per hour.

(LVI.) This example, and also (LVIII.), are intended to explain some circumstances of the motion of waters in estuaries and tidal rivers. It has been compiled from papers by Messrs. Frazer, T. Smith, and Mulvany, read at the Institution of Civil Engineers of Ireland.

The estuary of Ballyteigue, on the coast of the county of Wexford, which is shown in the Map, Fig. 78, lies at the back of a narrow range of sand-hills, stretching from east to west for about seven statute miles, and rising from 20 ft. to 60 ft. above the level of the sea at low water. At the western extremity of these sand-hills the estuary is connected with the open sea in Ballyteigue Bay by a narrow channel, which, at its mouth, is obstructed by a bar, composed principally of fine sand, and upon it, at low water of ordinary springs, the depth is about 12 inches.

The high-water line in the estuary is indicated on the Map by a strong full line, low-water mark by a dotted line within this, and the boundary of the lands that were liable to be flooded, by a fine dotted line outside of it.

At the entrance from the sea, the width of the channel, at low-water of ordinary springs, is 150 ft.; at high-water, about 300 ft. Up to the site of the embankment (the innermost of the two represented, and marked on the section "Embankment proposed by the Commissioners"), a distance of about 3¾ miles from the bar, the average width at low-water is 400 ft., increasing to 1400 ft. at high-water. Above the embankment, and on to Ring's Nook, 6¼ miles from the bar, the low-water channel is not more than 60 ft. wide, but the space covered at high-water of ordinary springs suddenly enlarges to nearly a mile in breadth, forming the Ballyteigue Lough.

At Ring's Nook the high-water channel is again contracted to about 900 ft., but it immediately widens again, and forms the "Broad Water," which, though very shallow, is a permanent lake, the tidal and fresh water being prevented from flowing out by a bar across the channel at Ring's Nook.

The northern extremity of the Broad Water is about 8½ miles from the bar, but the influence of the high spring tides extends up the river towards Bridgetown, about 1 mile further.

Thus we find this remarkable estuary to have consisted of, first, the narrow channel communicating with the sea, and nearly 4 miles in length; then a sudden expansion for about 2 miles, above which it is contracted for the length of a few yards, and again expands for a length of about 2 miles, and in a direction nearly at right angles to the former.

The extent of the whole basin, filled at high water of spring tides, is about 2200 statute acres, of which 600 lie between the bar at the bay and the embankment now constructed. The total area draining into the estuary, and discharging into the sea by the channel, is about 6000 acres.

The rise of ordinary spring tides in the bay is 11 feet, of neap tides 6 ft., and of extraordinary springs about 14 ft.

The study of the map of the estuary will, however, lead us to suppose that the peculiar form of the Lough caused a great difference in the time and height of tides within, as compared with their level and time outside—a large expanse of 2200 acres having to be filled and emptied through a very narrow and long entrance at the bar; and careful observation confirmed this surmise.

The section, Fig. 79 (the vertical scale of which is greatly exaggerated compared with the horizontal, being 352 to 1), is taken from the bar along the line of the main channel, and the line of the narrow channel through Ballyteigue Lough, over the second bar at Ring's Nook and through the Broad Water up the river course: on it are drawn, at the left-hand side, outside the bar, horizontal lines, showing the level of high-water of spring tides on the 30th May, 1843, which, from its agreement with other observed tides at the same place, was taken to represent fairly the "Establishment" of the bay, and the ordinary low-water of same, the latter being made the datum of the section, and also a line representing the actual low-water of the 30th of May, which was, however, kept up by high south-west winds. Two other lines are drawn within these limits, the higher, the level of high-water of neap tide 21st of June, 1843, and a dotted line which is the level of true neaps.

As we follow these lines inwards from the bar, however, we perceive the extraordinary fact that the high-water line is curved downwards from the bar, and the low-water line upwards from the same point; in fact, the capacity of the receiving basin was so great compared with the entrance, that the water of the flood-tide had not time to rise in the Lough up to the level of the tide in the bay; and at the ebb in the same manner, at low-water, it had not had time to flow out and descend to the level of the low-water line outside.

On the 30th of May, 1843, it was high-water (spring tide) on the bar at 6 A. M., the whole rise of the tide being 10 ft. At the under-named places, from the bar inwards, the time and height were as follows:—

	Time.	Distance.	H. W. *lower* than that at the bar.
The bar of Lough, . . .	6 A. M.	0	0
Coast-guard Station, . .	6 „	300 yds.	18 in.
Point, 2 miles from bar, .	6 „	2 miles.	21 „
Ring's Nook, ...	$8^h\ 8^m$	6¼ „	32 „
On the Broad Water, .	$9^h\ 0^m$	7½ „	36 „

It thus appears that it was half ebb at the bar when it was high-water in the Broad Water; the tide continuing to flow into the Lough after it had begun to descend in the bay.

The low-water of the same day is thus recorded:—

	Time.	Distance.	L. W. *higher* than that at the bar.
Bar of Lough, . . .	12^h 8^m P. M.	0	0
Coast-guard Station, .	12 25 ,,	300 yds.	15 in.
Point, 1¼ miles from bar,	1 0 ,,	1¼ mile.	30 ,,
Site of proposed embankment, ...	2 0 ,,	3¾ ,,	30 nearly.
Ring's Nook, ...	3 25 ,,	6¼ ,,	52 in.
Broad Water, ...			72 ,,

We may, without much error, suppose the rate of rise of tide in the bay to be nearly uniform, rising *uniformly* for six hours, and ebbing in the same manner, and for the same period.

It appears from the above tabulated observations, therefore, as was beforesaid, that the water of spring tides had not time to enter fully into the estuary in the six hours of flood; nor again, had that which did enter sufficient time completely to flow out during the six hours of ebb.

The irregular form of the estuary, and of the channel connecting it with the bay, renders it impossible to make precise calculations of the extent of area which, with a given entrance, would be exactly filled and discharged in twelve hours; it will, however, be shown generally, that there is a relation between the vertical transverse area of the entrance and the horizontal area of any receiving basin of a uniform depth, and having vertical sides.

During neap tides the effects in the estuary were very different from those observed during springs. The heights of high-water, and the depths of low-water, as well as the time of flood and ebb, coincided more closely with those in the open sea. This is occasioned by the fact that the neap tides never enter the Broad Water above Ring's Nook, nor rise over the extensive mud banks of the Ballyteigue Lough; so that, with the exception of the narrow channel up to Ring's Nook, we may look upon the neap tide basin as terminating at the site chosen by the Commissioners for the embankment intended to exclude the tidal water. The following observations on the neap tides have been selected:—

	Distance from bar.	*Lower* than H. W. at the bar.
At site of proposed embankment, now constructed, ...	2 miles.	7 inches.
At Ring's Nook, ...	6¼ ,,	13 ,,

instead of 21 inches and 32 inches, which were observed at springs. The low water of this neap tide had therefore an average inclination of about 2 in. per mile falling from Ring's Nook to the bar. It must be observed that the lines upon section, Fig. 79, showing the high-water and low-water of springs, do not represent the surface line at any one moment of the ebb and flood within the bar, but are merely the lines joining the points of high-water mark at the several successive distances from the bar.

Several different plans were proposed for the reclaiming of this Lough, two of which are shown on the map, Fig. 78. One, by Sir J. Macneill, took up the streams flowing into the Broad Water by a new river course through it, passing Ring's Nook, and along the eastern boundary of Ballyteigue Lough, discharging into the sea by cutting through the sand-hills near Crossfarnogue Point. The tidal waters being intended to be kept out of the lough by an embankment shown on the map and section, the westernmost of the two indicated, and designed at a very narrow place, but deeper than that to the right, selected by the Commissioners; their plan, which was that finally carried out, took up the different streams flowing into the lough by a navigable river course, which, commencing near Bridgetown and passing inland, reaches the Duncormac stream, down which it proceeds to its junction with the entrance channel, being indicated by a thick dotted line. This plan had the double advantage of providing for the local navigation, and also admitting a larger volume of water each tide up to the main embankment. Another plan proposed was to construct an embankment on each side of the existing low-water channel through the lough. The disadvantages of this method will be made clear by the study of (LVII.).

All designs which involve the construction of a main embankment across the natural channel require an excavated reservoir on the inner side, and self-acting sluice-doors in the embankment: the one to receive all the rain-water, springs, leakage, and percolation that can accumulate upon the reclaimed lands during the period in which the action of the tide outside keeps in this water; the other, to discharge it during the interval in which the relative levels outside and inside admit of its flowing out.

The plan finally adopted for the reclamation of this estuary was, the construction of an embankment distant about 3 miles from the bar, shown on the map, Fig. 78, being the point at which the entrance channel may be considered to terminate, and the wide flats, covered at spring tides, to commence, sluice-doors of the requisite area, and placed at a sufficient depth, being constructed in it.

These sluices have to discharge at low-water all the rainfall upon a considerable district draining into the lough, which is allowed to collect during each tide at an excavated reservoir on the east or land side of the embankment.

Another work, rendered necessary by the reclamation of the lough, was the formation of a navigable channel, shown on the map, which also acted to convey the waters of the chief stream, formerly flowing into the sea, through the area now laid dry; but for this channel not only would the water traffic of the locality have been destroyed, but also the sluices in the embankment would necessarily have been greatly increased in number and total area, and a larger reservoir been required.

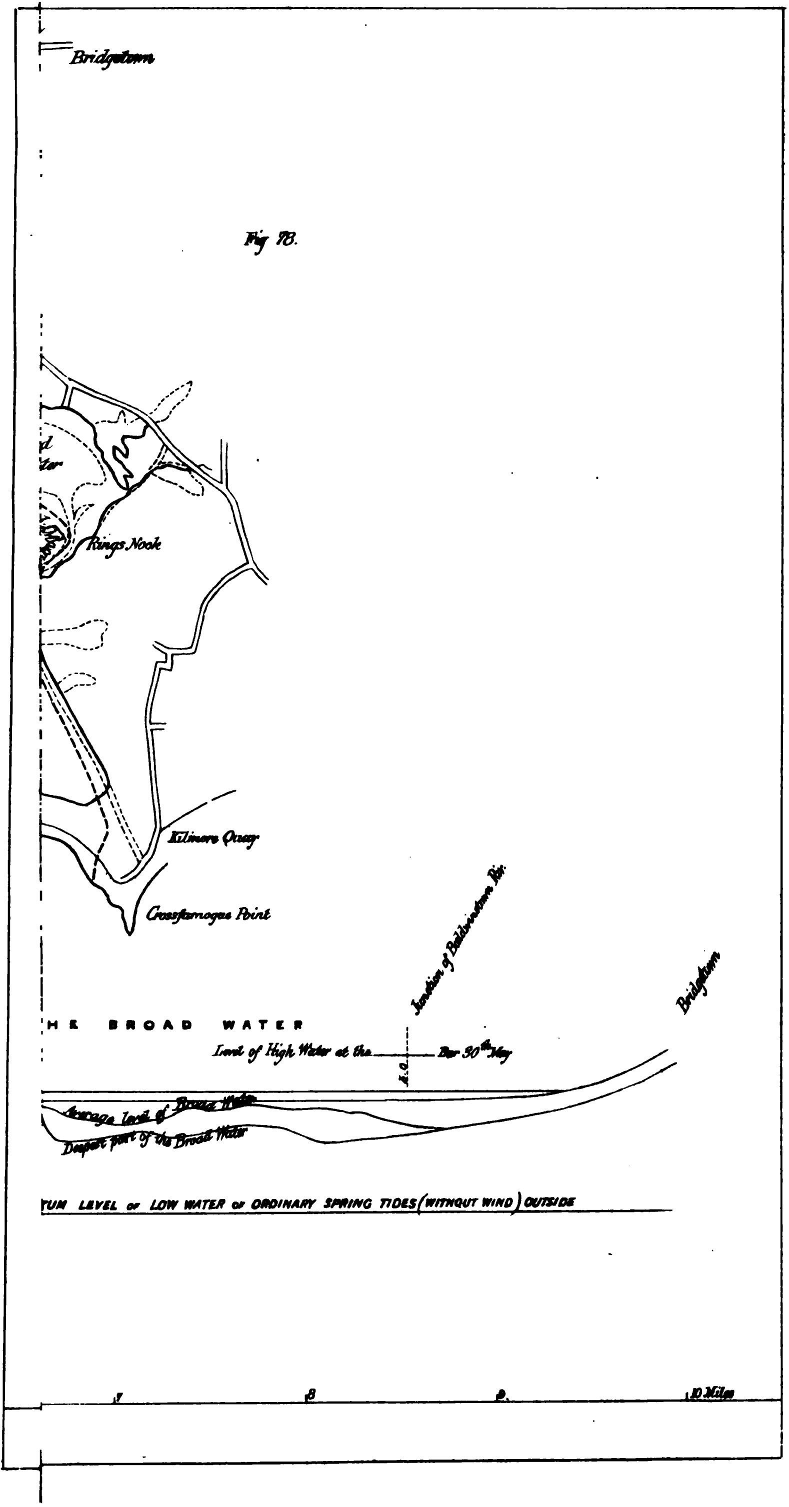
As a preliminary, and with a view to facilitate the future construction of the main embankment, and consequent final exclusion of the tide, a subsidiary embankment, to exclude the sea from the Broad Water, was completed at Ring's Nook, 3 miles further from the bar than the main embankment above mentioned. The results of this work at Ring's Nook were very encouraging, and showed conclusively that the cause assigned for the peculiarities in the surface of the tide within the estuary was the true one.

It was ascertained by accurate levelling, that at Ring's Nook, after this lesser preliminary embankment had been completed, the high-water mark of springs was now only 18 in. lower, and low-water mark but 25 in. higher than that at the bar. So that the former was raised 14 in., and the latter depressed 27 in. compared with the observations on the 30th of May, 1843, p. 157—the range of the spring tide that day being but 3 ft. at Ring's Nook, but on this occasion 6 ft. 5 in.

On the completion of the main embankment, at the point indicated on the map and section, and the total exclusion of the tidal water, no difference could be observed between the level of high-water of either spring or neap tides, so that the injury to the navigation, anticipated by some persons as likely to arise from reclaiming the estuary, resulted in a positive advantage, in so far as an increased depth at high-water was given.

The Commissioners, it is true, had to bear the expense of alterations at the Coast Guard Station, and had also to raise an embankment, previously constructed by private parties, parallel to the course of the stream from Duncormac, by which some land had been reclaimed.

Fig 78.


Bridgetown
d
ter
Rings Nook
Kilmore Quay
Crossfarnoges Point
Junction of Baldwinstown Riv.
Bridgetown
HE BROAD WATER
Level of High Water at the Bar 30th May
A.O.
Average Level of Broad Water
Deepest part of the Broad Water
TUM LEVEL OF LOW WATER OF ORDINARY SPRING TIDES (WITHOUT WIND) OUTSIDE
7
8
9
10 Miles

All these having been constructed with reference to the former high-water mark, which was justly looked upon as a natural and unalterable point, but in an unexpected manner they had to be adapted to the altered state of things. Nor was the depth upon the bar lessened from any diminution in the total volume of the back-water discharged each tide, with a scouring action; the increased rise of the surface up to the main embankment, which now constitutes the furthest limit of the influx of the tidal water, coupled with the increased depth at low water, giving very nearly an equivalent for that which formerly passed up beyond this point and spread over the flat mud-lands of the Lough, and occasionally into the Broad Water.

It was computed that, in the original state of the estuary, the quantity that flowed in during spring tides was about 163,000,000 cb. ft., of which 78,000,000 was between the bar and the main embankment, and 85,000,000 above its site; now, by the increased rise and fall above mentioned, 68,000,000 are added to the 78,000,000, giving a total, up to the embankment, of 146,000,000 each spring tide, only less than the original quantity by 17,000,000. It was, therefore, with a sound judgment that this position of the embankment was decided upon, admitting a much larger volume of water than that which was proposed to be placed nearer the bar, standing also, as appears from the section, upon much higher ground for its base, and immediately before the point at which the channel opens out into the wide level now reclaimed.

In the construction of the sluices for the discharge of the land water, both as to their density, their figure, and the depth at which they were placed, we may trace some good practical points of construction. The soffit, or upper line of the openings for the discharge, was placed 18 in. beneath the line of lowest water; so that the whole area of the sluice-way was always completely immersed, and the sluice-door itself was so constructed that the smallest hydrostatic pressure from the inner side, resulting from the external level descending ever so little below that in the reservoir within, immediately removed it from the whole of the area of the aperture.

The Figs. 79*a* and 79*b* represent the elevation and side view of one of the four self-acting sluice-doors by which the reclaimed lands are unwatered, at a scale of $\frac{3}{10}$ in. to a foot, or $\frac{1}{40}$. The bearing frame of the door is represented at ABCD, composed of iron, and rendered rigid by the cross bars E, E; it is capable of motion upon the axle FG, which passes through the eyes of the anchor irons H, H, H, firmly fixed into the masonry. At a distance of about 2 ft. 4 in. below FG is the axle MM, which is the working axis around the upper line of the surface of which the door revolves in opening and closing. NNN are strong iron stems, firmly connected with the straps of the door, having eyes forged upon their upper ends, by means of which they revolve upon the axis MM. In the side view is shown, at S, a stop forged upon the outer face of the stems, and set at such an angle with the axis of the door as to prevent its rising higher than the position shown in the dotted lines, Fig. 79*b*, in which the door is completely clear of the upper line, or soffit, of the aperture.

The axis MM is placed at such a level as to be always immersed at low-water; but FG is accessible at low-water of all tides, so that the door can be taken off at any low-water if required; a ring-bolt, shown at R, is provided to assist in raising in it, the door itself, and the frame by which it is suspended, in that case turning around the axle FG. The total area of the four openings was 96 sq. ft., the door overlapping on every side about 4 in. Stop-gates were provided, so as to admit of complete access in case of repairs.

In order that the sluice-door may rise the instant the external pressure is in the smallest degree withdrawn, it is necessary, first, that the specific gravity of the compound mass of iron and wood of which the door consists should be equal to that of water, and secondly, that the centre of gravity of the sluice-doors be a little above its centre of buoyancy. To give full effect to the first condition, it is necessary that the sluice-door, when in the position shown in the dotted lines, Fig. 79*b*, floating completely clear of the upper line of the aperture, should yet be fully immersed in the water.

The materials of which the door consists are iron and elm timber (the latter having great durability under water). The specific gravity of iron being 7645, of elm 671 to water 1000, we must, to fulfil the first condition above, calculate the bulk of each, so that the specific gravity of the door be 1000. If we put I, E, and C for the weights of the iron, the elm timber, and the compound of both forming the door; also, i, e, and c for their specific gravities respectively, and, remembering that the weight divided by the specific gravity in any body equals its volume, we have

$$(1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad I + E = C,$$

and

$$(2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \frac{I}{i} + \frac{E}{e} = \frac{C}{c}.$$

As it was easier to adapt the bulk of the timber to that of the iron, the mode of proceeding adopted was this:—from the drawings of the design of the sluice-doors, a complete set of all the iron work of one of them was constructed, which, on being accurately weighed, were found to be 272 lbs. The problem then resolved itself into this, what amount of dry elm combined with 272 lbs. of iron, will form a compound structure of the same specific gravity as water?

We have from (*a*), $E = C - I$, substituting this value of E in (*b*) we obtain after a few reductions,

$$C = I \times \left(\frac{i-e}{c-e}\right) \times \frac{c}{i};$$

therefore,

$$C = 262\left(\frac{7645-671}{1000-671}\right) \cdot \frac{1000}{7645} = 727 \text{ lbs.};$$

from which we must, to obtain the weight of the timber, deduct the weight of the wrought-iron straps, &c., or $727 - 262 = 465$ lbs., giving the weight of elm, which, at a specific gravity 0.671 gives very nearly 11 cb. ft., or $11 \times 1728 = 19008$ cb. in., and this divided by the area of the sluice-door, 8 ft. 8 in. × 3 ft. 9 in. = 4680 sq. in., gives the required thickness, namely, $19008 \div 4680 = 4.06$ in. The doors were constructed $4\frac{1}{8}$ in. in thickness, or 4.125.

The second condition for a self-acting sluice-door, namely, the distribution of the density, so that the centre of gravity of the compound mass should lie above the centre of pressure of the volume of water displaced, was fulfilled by a small modification of the weight of iron in the stems N, N. Fig. 79*b* gives the several distances CG and CB to the centres of gravity and of buoyancy respectively, namely, 32.625 and 35 in., thus the former is 2.375 in. higher than the latter, and hence the *instability* of the door when vertical, which causes it to rise instantly on the smallest lateral pressure from within.

These sluices have been quite successful in practice, fulfilling all the objects aimed at in their construction, and are perfectly self-acting. To test them, graduated gauges were placed, one in the front, the other inside the sluices, and reading from the same zero; it was found that the instant the tide water outside falls to the smallest degree below the level of the drain-water accumulated inside, the doors fly up to the position shown in dotted lines, Fig. 79*b*, and upon the flood-tide making, they close with almost equal facility, thus affording the maximum discharge by rendering available the whole period in which the relative levels admit of the water flowing out.

The object of the reservoir already mentioned will be understood, by considering the large proportion of the whole time of the rise and fall of the tide that the self-acting sluices must remain closed, and the fact, that the water thus impounded inside the embankment must not be allowed to rise so high as to injure the reclaimed lands under cultivation. In order to fix the ideas; let us assume a locality where, as at Ballyteigue, the rise of neap tides is 6 ft., and also suppose this rise and fall to be at a uniform rate, which is not far from the truth. Neap tides are taken as being less favourable to the discharge than springs, which fall as much lower as they rise higher than the neaps.

In sluices of good design, the level of the water at the end of the ebb will be the same on the outside as on the inside of the sluices in the reservoir, and at the first of the flood-tide the self-acting doors will immediately close, and remain shut during the six hours of its rise, plus such part of the six hours of ebb as may elapse until the surface has fallen down to the level at which the inner drainage-water now stands; this last having been continuously rising during both the above periods of time; and when this, the uppermost common level is attained, the doors open, and the discharge commences, and continues until dead low-water is reached.

If, then, we have as data the height of the lowest part of the reclaimed lands above low-water, and its necessary elevation above the highest surface water of the reservoir so as to be uninjured, which must be about 2 or 3 ft., we can determine the height of the highest level that the water in the reservoir should be permitted to rise. Thus, at Ballyteigue, the lowest lands were stated to be about the level of high-water of neaps, so that 2 ft. below this, in order that the land may be uninjured, gives 4 ft. as the greatest rise of the inner waters; and, consequently, the sluice-doors are closed for 8 hours,—namely, 6 hours of flood and 2 hours of ebb, the last answering to 2 ft. at a uniform rate of rise and fall, so that we would have, with these particular data, the sluices shut and drainage-water accumulating for 8 hours, and fully open and discharging during 4 hours, and the range of surface of the reservoir 4 ft.

If then the volume of drainage-waters which collects in 8 hours be computed, we obtain the superficial area requisite by dividing this volume by the range, 4 ft. Take the rain-fall as 1½ in. in 24 hours, and that ⅔ of it, or 1 in., is ultimately delivered into the water-courses, we have ⅓ of an inch in 8 hours; and supposing the total area that drains into the reservoir as 3000 acres, we have this proportion

4 ft. (or 144 thirds of an inch) : $\frac{1}{3}$ in. : : 3000 : 20.8,

nearly 21 acres; but if the main drains through the lands are properly formed for such place, they will constitute a large part of this area, they should be excavated their whole length back at the same level as the bottom of the reservoir, which must be somewhat below the line of low-water of lowest spring tides. The very small surface inclination that the main drains are found to work with, which is about 3 in. per mile, shows the importance of having the sills of the sluices as low as possible; if placed 1 ft. too high this would destroy the drainage of 2 miles in the distant part of the district. It will be sufficient if the soffit of the aperture be as low as low-water of springs, and its area be large enough to bring the water of the reservoir down to the level of that outside.

At Ballyteigue, the soffit was placed 18 in. below lowest low-water, in order that the whole of the sluice-door might be at all times immersed; and, it is presumed, in order that the inclined position in which it floated might facilitate its closing, by offering a surface for the action of the flood tide, the current inwards pressing it against the masonry and shutting it.

The study of the remarkable features of this estuary suggests the question,—What, if any, is the relation between the horizontal area supplied each tide, and the vertical transverse area of the entrance channel through which the tide ebbs and flows, so that the level of the water in the estuary may be the same as that of the sea at high-water? It is necessary to suppose the entrance to be rectangular, and the inner basin prismatic in form, that is, with vertical sides, in order to give the question sufficiently simple conditions. We may then use Hutton's* equation, between the height of the tide x, supposed to rise uniformly, and the height z, that the water will rise, in a given time, in a basin of an area A, the width of the entrance being equal to b, to determine, as in this case, the area, when x and z are equal and known, namely, 10 ft., the rise of the spring tides.

This relation is

$$z = \frac{4}{5m} \times x^{\frac{5}{4}},$$

in which

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* Tract 38, Problem xxx., Hutton's Mathematical Tracts.

$$m = \frac{A}{2160'' \sqrt{16.08} \cdot b}$$

in the case of a uniform rise of 10 ft. Now, if we make $z = x$, we find

$$A = 218{,}494 \times b,$$

that is, with 10 ft. rise,

$$A = 21{,}850 \times 10 \times b;$$

or the area of the basin must be not more than 21,850 times the area of the entrance channel; in other words, for every square foot of transverse area in the entrance channel we must have half an acre in the basin. But the rise of the tide is seldom uniform, and at this locality the entrance channel had sloping sides, and was 4 miles long, instead of a mere rectangular opening, and the basin of very irregular shape instead of being prismatic, as in the problem. In the Minutes of Proceedings of the Institution of Civil Engineers, vol. ix., in the description of the Great Grimsby Coffer Dam, particulars are given of the flow and ebb of the tide through a rectangular opening 20 ft. wide, and a note with calculations by Mr. Beardmore.

(LVII.) The Ganges Canal, which is represented in the sketch map, Fig. 80, at a scale of 75 miles to an inch. is one of the earliest completed of the great works of irrigation undertaken in Hindostan, and supplies an instructive example of the various considerations involved in designing the works for the conveyance of very great volumes of water. And first, of the physical features of the district. The Ganges, having its sources high up among the Himalayas, leaves its mountain channel above Hurdwar, Figs. 80, 81, at which point it escapes through the Servalie, or lower Himalaya range, into the plains, a pure, rapid, and plentiful stream, flowing over a bed of large boulders, where it separates into several channels, one of which, Fig. 81, passes the Pyree Ghat, and by this channel the supply for the canal is brought to a point a mile below Hurdwar.

The district irrigated by this canal is situated between the great rivers Ganges and Jumna, which run in a direction nearly parallel to each other, towards the south-east, meeting at Allalabad, not very distant from the limit of the map.

The whole of this great tract has a general slope from the foot of the lower Himalayas in the same direction as the course of these rivers, downwards to their point of junction. It was necessary, therefore, with a view to irrigate it by gravitation, to convey the canal of derivation to a point sufficiently high to command the whole district which is found contiguous to Roorkee, Fig. 81; and hence the reason of the choice of Hurdwar, upon the Ganges, as the locality from which to draw the supply. Referring now to the enlarged map, Fig. 81, which has a scale twenty times larger than Fig. 80, we may trace the course of the canal for the first 21 miles. At the point already mentioned, a mile below Hurdwar, the excavation commences. Here a regulator and an escape-dam 580 ft. long is constructed, both provided with gates, so that any quantity of water can be sent down the canal, or the supply cut off at pleasure. From this point the canal, which has a bottom width of 150 ft., and depth of water of 10 ft., by taking a westerly direction, crosses all the hill drainage at right angles; and on arriving near the Ratmoo river, by a sharp curve, flows nearly south, crossing the Ratmoo and the Solani, two of the largest torrents. Here the high land of the Doab is reached, and the termination of the first or northern division of the canal.

It was, no doubt, a bold step to convey the canal across torrents, one of which discharges upwards of 100,000 cb. ft. per second, when in high flood; but the result has justified the attempt. The levels, however, would not admit of the head of the canal being situated below the confluence of the Solani with the Ganges.

In determining the present line, the slope of the canal-bed had chiefly to be considered. This was a problem of the most difficult nature to solve, namely, given a volume of 6750 cb. ft. per second to be discharged, to find what was the best transverse section and inclination per mile, to convey that body of water at such velocity as, on the one hand, not to tear up the bed, and on the other, to prevent the vegetation, which would arise from its being too slow. The section determined on was, bottom width 150 ft., with an inclination of 15 in. per mile. This slope, it is believed, has since been found rather too great, and 12 in. would probably have been sufficient; an alteration which may be effected by raising the invert or flooring of the bridges, without interfering materially with the navigation.

This is a point of much greater importance than might at first sight be supposed; for on it depends not only the stability of the works, but also, in a great measure, the beneficial effects of the canal. For, according to the velocity of the water over the canal bed, so does the abrading power vary, and also the nature of the particles of earthy matter brought down by the stream, which must all eventually be spread over the fields. If, therefore, the soil becomes covered with a stratum of sand, the canal instead of being a blessing to the country, may prove otherwise.

The slope of the country, for the first 20 miles of the canal, rather exceeded 3 ft. in the mile, an inclination far too steep to be used in conveying so large a volume of water. The difficulty was overcome in the same manner as it had already been in the reconstruction of the Eastern Jumna Canal,—shown in the map, Fig. 80, above Delhi,—namely, by introducing transverse steps in the line of the bed at certain points, analogous to those which may be perceived in every mountain stream in nature. A reference to the section, Fig. 82, will explain this: in it are seen four nearly vertical steps of 9 ft. each, and one of 8 ft. at the extreme left of the section, which represents the first 21 miles of the canal. These falls are massive structures, bridged over, and having a profile rounded off at top, and hollow below, like an "ogee" curve, having a masonry apron in the rear, 70 ft. broad, protected by piles and boxes filled with boulders.

From the great dimensions of the canal, 10 ft. deep, and 150 ft. wide at the bottom, and its necessarily moderate velocity, it was evidently adapted to navigation in both directions. To admit, therefore, of boats passing these rapids or falls, a separate navigable channel, provided with locks, leaves the main channel half a mile above the fall, and again joins the canal half a mile below. Two are shown on the map, Fig. 81: one on the east side of the Ranipoor torrent, passing the first fall, and the other on the west of it, which passes the second, third, and fourth falls, both of these being on the south side of the canal; a third is shown at the fifth fall, below Thomason College.

As the direction of the first 20 miles of the canal is transverse to the general inclination of the land, and cuts the streams at right angles to their course, provision must be made for their uninterrupted discharge, and, as in the case of railways and roads, this may be effected by either sending them under the canal, or over it, or using what may be called the level crossing. The works for this last consist of an inlet, escape-dam, and regulator drop-gates across the canal. The inlet is for admitting water into the canal when the flood is so high as to force a passage for itself; and the gates are so constructed as to open of themselves when the pressure on the up stream side of the gates becomes greater than the pressure exerted by the water in the canal to keep the gates shut. When this takes place, the sluice-gates on the escape are opened, and the drop-gates on the regulator are allowed to fall through grooves prepared to receive them, so that no water is permitted to flow down the canal during the flood. Thus the whole volume of water flowing down the canal, along with that of the river, are discharged over the dam, and escape together down the bed of the stream. Had any of the flood water been allowed to flow down the canal, so great is the proportion of earthy matter brought down by these torrents, that the canal-bed would soon get silted up.

The floods, however, seldom continue more than five or six hours, and do not occur oftener than eight or ten times in a season, so that the inconvenience of cutting off the whole supply is only temporary. The average rain-fall in this part of India is stated to be 42 in.; but it is not, as in this country, diffused over every month of the year; the greater portion of it falls between June and September, and 4 or 5 in. in the cold season soon after Christmas. On some occasions 13½ in. have fallen in 8 hours, and even 3 in. in 35 minutes is recorded. With an inclination of 25 or 30 ft. per mile for such torrents, we can understand the difficulties of rendering the works of the canal secure.

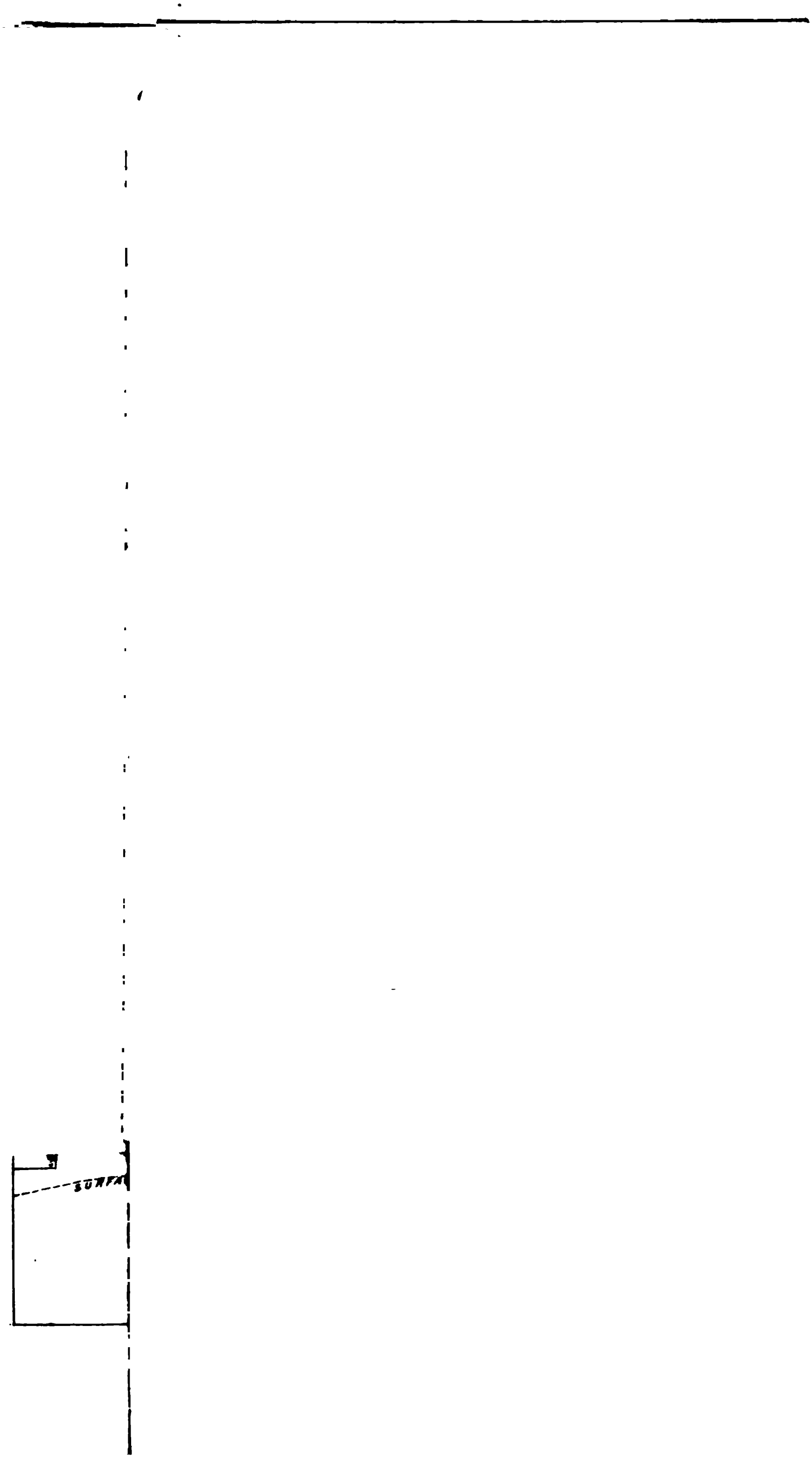
On the section, Fig. 82, are seen three inlets and escapes of 50 ft. length, one of 100 ft., one of 150 ft., and that of the Ratmoo river, the inlet and escape of which are each 800 ft. in length, with piers 10 ft. apart, and 10 ft. high, with sluice-gates attached, which are made to fall down level with the canal-bed when a flood is escaping, which sometimes rises 3 ft. above the top of the piers. The regulating-bridge has ten water-ways, each 20 ft. broad, and provided with drop-gates to prevent any flood-water from passing down the canal. The whole of this work is protected by a forest of piles, and an enormous number of bottomless boxes filled with boulders. The apron of box-work in rere of the escape is 100 ft. wide.

The river, when not in flood, passes under the canal by a double tunnel, upwards of 500 ft. long, thus preventing the inundation of the valley to the extent of nearly a square mile, where the country is below the level of the surface of the water in the canal.

It had been, at the first, intended to deal with the Ranipoor and Puttri torrents by inlet and escape; but by a well-designed alteration in the position of the contiguous falls or steps in the bed of the canal, which, from being below the point at which the torrent crosses, are now brought just above it, so that the bed and surface-water of the canal is lowered 9 ft., which admits of its being passed *under* the river, for which a super-passage has been constructed,—that for the Ranipoor of 300 ft. water-way, with high parapet walls, and for the Puttri, 200 ft.; the canal below having eight water-ways, each 25 ft. wide, and a separate water-way for the navigation, with locks.

Lastly, by the aqueduct of Solani the canal is carried over this river, which is computed in floods to discharge 100,000 cb. ft. per second. The aqueduct has fifteen arches of 50 ft. span, the length between the abutments is 890 ft., and width between the parapets 190 ft. The valley of the Solani is about 3 miles wide at the crossing of the canal, the bed of which is 16½ ft. above the line of the country, and formed by an embankment of earth, the sides being protected by a double line of revetments. On emerging from the excavation on the south side of the valley, the canal reaches the high land of the Doab about the twentieth mile of its course. All these works, embracing the chief difficulties, and nearly half the entire cost, were but means to an end, that is, the conveyance of the water, taken up at Hurdwar, to a point commanding the high land of the Doab; the remaining 870 miles of its course runs parallel to the drainage of the country; therefore, by following the watershed line of the district between the Ganges and Jumna rivers, and their tributaries, which drain the Doab, the canal is made to flow along the "backbone" of the country without intersecting any cross drainage, and also the water can be drawn off for irrigation to every point of the Doab. Branch lines are also given off at either side, immediately above the sources of those rivers which drain the country, and their courses are directed on the same principles as the main trunk: this will readily be understood by the study of the map, Fig. 80, and the sketch section on the line AB. The three irrigating canals are indicated by the marks on the summits or watersheds, the natural drainage flowing in the several valleys.

As in the first 20 miles, so here, falls or steps are built at various points along the line, for the slope of the country at the upper part of the Doab is somewhat in excess of that of the canal-bed; commencing with an average slope of 2 ft. per mile, it gradually decreases, until it is even less then 12 in. Altogether there are sixteen of these falls on the canal. The irrigation-channels are supplied from 282 heads, constructed of masonry, at about every 3 miles along the course of the main channel, which they leave at a given angle with the stream, so as to insure a uniform discharge down all the irrigating lines. Thus the main channel of an irrigating canal,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unlike a river, is largest at its source, and diminishes in volume as it approaches its point of final discharge; the volume of water flowing down the lower portions of the terminal lines being about one-twentieth of what it is at the head of the Ganges Canal. The navigable channel of this canal bifurcates at the 180th mile, near Nanoon, into two navigable channels, one terminating at Cawnpore on the Ganges, and the other joining the Jumna, below Etayah, at which point the bed of the canal does not exceed 20 ft. in breadth. The arrangement for obviating the too great fall in the bed of a channel, if constructed identical with that of the country it passes through, by a series of falls or steps, as in the Ganges canal, may be found in the works of the late Mr. Thom of Greenock. It is also applied on the New River from Ware to London, and other hydraulic works.

This account of the Ganges Canal has been compiled from a paper by Mr. T. Logan, C. E., published in vol. v., part 1, of Transactions of Royal Scottish Society of Arts.

A statement of the discharge of some of the chief rivers of the British Isles, compared with that of the Ganges Canal, may serve to give a further idea of its magnitude. The canal draws, as has been said, 6750 cb. ft. per second from the river, which is equal to 405,000 cb. ft. per minute. Now the Shannon, at Killaloe, in the driest summers, has been estimated to discharge 100,000 cb. ft.; in greatest floods, 1,000,000 cb. ft. per minute. The ordinary summer discharge of the Thames, at Staines, is about 40,000 cb. ft. per minute; of the Severn, at Stonebench, 33,100; and of the Nene, at Peterborough, 5000 cb. ft. per minute. The author above-named states that the canal is equal to the discharge of the Tay, the Forth, and the Clyde, all put together.

It might, indeed, be doubted if even the Ganges would in the dry season invariably afford this immense volume of water, and that too at a time when it was most essential for the irrigation of the territory dependent on it, about 5,400,000 acres, which is one-half larger than Yorkshire; but this river, like others rising in mountains so lofty as to be covered with perpetual snow, comes down in great strength from the melting of the glaciers and snows, even when no rain may fall, and the sun parch up the lowlands. The same remark applies to the value of the river Durance for irrigation (LIX.), which takes its source in the glaciers of the western slope of the Alps, and is fed by the melting of the snows in summer.

Capt. Baird Smith remarks upon the superior value of the tributaries of the Po which flow in from the north, compared with those from the southern slopes: the latter, rising in the Apennines, which are below the level of perpetual snow, become dry in the summer season; the former, rising in the Alps, on the contrary, afford an unfailing supply to the best irrigated district in the world.

(LVIII.) Nearly 800,000 acres on the eastern coast of England, comprising part of the counties of Lincoln, Cambridge, and Huntingdon, form what is called the Fen district, and being nearly on a dead level, and only from about 8 to 14 ft. above the line of low-water at sea,—in many parts, indeed, actually below high-water line,—are drained with great difficulty. Several considerable rivers, as the Welland, the Nene, and the Ouse, &c., which have their sources in the high land surrounding the Fens, traverse the intervening level district to their outfalls at the sea.

The available fall in this latter part of their course is extremely small, so that, in improving the natural circuitous channels, it is found essential to provide for the due discharge of the water by the superior straightness of their courses, and large area of their transverse sections. The rate of fall that can even thus be obtained is but a few inches per mile. Telford and Rennie have completed some successful works on a vast scale in this interesting district: that known as the Eau-brink Cut will afford, perhaps, the best example of the importance of a direct compared with a circuitous course in the outfalls of this district.

The map, Fig. 83, shows the old and winding course of the river Ouse, and the New or Eau-brink Cut, that now conveys the water, and which was completed in 1821. The longitudinal section is shown in Fig. 84, and below it three transverse sections at the points indicated, which give a width at waterline of high-water of spring tides of 412 ft., and depth of 26 ft. 3 in., and at low-water 312 ft., and 11 ft. 9 in., taken just below Free Bridge. The gain in distance is from upwards of 5 miles by the circuitous course of the former natural stream, to 2½ miles, the length of the new cut from Eau-brink across the marshes to the town of King's Lynn. The datum of the section is the sill of Denvir Sluice, which discharges the water of more than 60,000 acres, and is situated up the course of the Ouse, about 9½ miles above the commencement of the Eau-brink Cut.

The effect of the improvement of the channel has been very remarkable: thus the average depth of low-water on the sill of Denvir Sluice was previously 9.6 ft., now it is about 6 ft. less, or 3.6 ft., and at the commencement of the cut it is from 8 to 9 ft. lower than formerly. The high-water mark has, on the other hand, risen at Denvir Sluice from 11.83 ft. to 13 ft. Moreover, the new channel, as is shown in the longitudinal section, Fig. 84, and the three transverse sections, has, since its opening, excavated its bed from 8 to 10 ft. by the scouring action of the flow and ebb of the tide; and not only in this part of its course, but between Eau-brink and Denvir Sluice; also, the bed of the river has been lowered on an average 10 ft., and its general sectional area has increased about a third. By means of this lowering of the surface of the outfall channel at low-water, a perfect natural drainage has been afforded to extensive tracts of lands, previously but ill-accomplished by an expensive and complicated, but ineffectual, system of windmills and steam engines.

Notwithstanding that the navigable channel in Lynn harbour was found to be deepened 7 ft., after the completion of this work, yet at this place the low-water spring tides were still 5 ft. higher than in the roads at the entrance of the river, owing to the winding course of the channel below the town, and the prevalence of bars and banks of sand and mud; a part of which are shown upon the map, Fig. 83. An outfall is now, however, in progress from King's Lynn to the roads, of 4 miles in length, which will have the effect of bringing dead low-water up to Lynn, and of lowering the water at the end of the Eau-brink Cut 5 ft., and ultimately still further benefit the lands on each side of the Ouse.

The old course of the river has been embanked across, and warped up; this last being a simple operation, rendered feasible along this coast by the quantity of earthy matter in mechanical admixture with the sea-water of the coast; the tide being admitted until high-water, a sluice in the embankment is then shut, and the water retained until it becomes clear by depositing the mud, after which it is let off at low-water, and the process repeated with the next flood-tide, and continued until the surface of the land is raised sufficiently high for cultivation.

The inclination of the low-water surface in the river, previous to opening the cut, is shown on Fig. 84. Careful observations since its completion show that in great floods the average fall per mile between Denvir Sluice and Free Bridge, a distance of 12 miles, 5 furlongs, was nearly 9 in. per mile on the surface of low-water. In summer time the fall has been as low as 2.6 in. per mile.

The River Nene, which also discharges its waters into the Wash, and on its northern shore was, previous to 1834, like the Ouse, impeded in its outfall by a confined and tortuous channel, and by its high and shifting sands; the surface-fall at low-water for the last 4 miles having been 12 ft., or 3 ft. per mile; in the next 1½ miles up stream, the fall was 1 ft.; the next 11½ miles higher, the fall was but 1 ft. 6 in.; the total 17 miles giving a total fall of 14 ft. 6 in. It was evident, therefore, that the great impediment to the free discharge of the waters of the Nene, and of course to the general drainage of the Fens depending on it, lay in the last few miles towards the sea; Mr. Rennie, consequently, recommended the river in that part to be carried by a new cut, of suitable capacity, across the marshes to a point called Crab-hole, 5½ miles in length (for the maps and sections of this we refer to Life of Telford). This has been completed since 1834, and with results exceeding the most sanguine expectations; the surface-fall at low-water is now, for the last few miles, in ordinary states of the river, but 2 in. per mile, and in times of land-floods 4 in. per mile.

So confident was Mr. Telford, who was engineer together with Sir John Rennie in the Nene Outfall, of the success of the work, that, before its completion, he undertook the North Level drain, a tributary of this river, the cost of which was £150,000; and, as it opened into the Nene, its great depth would have been utterly useless, had the latter not been successfully lowered. The result was, that the new channel ebbs every day 10 ft. lower than it did in the old channel, immediately opposite to the South Holland and North Level sluices, which are the outlets for 100,000 acres of fen lands lying between the rivers Nene and Welland towards the north, and superseding an expensive and complicated, but ineffectual, system of windmills and steam engines.

Among other beneficial results of the Nene Outfall, there is now a safe and daily communication by water between the town of Wisbeach and the sea, at all periods of the tides (both spring and neap), and in all weather, in place of the navigation through the old channel, which was tedious and dangerous, and capable only of affording a passage at spring tides, and with a favourable wind, to vessels drawing at the utmost 6 ft. of water, and carrying 60 tons. The spring tides, which formerly rose but 4 ft. at this town, now rise 13 ft.; and neap tides, which some years ago did not reach the town, now rise 9 ft.; the trade of the port, which before the completion of the Nene Outfall was not more than 50,000 tons, has since reached 108,000 tons per annum.

This incidental effect of straightening and deepening the outfall of a tidal river, has been made the chief object of such operations in many parts of England, in which the drainage of the contiguous lands is not required. The Dee, the Ribble, the Clyde, and the Tay, may be instanced, and many others (*vide* "Beardmore's Tables," second edition). In these, the improved channels admit the tidal wave with less resistance, and both the time and height of high-water more nearly approach those of the open sea: the regular inclination, easy curves, and uniform lines, and transverse sections of the artificial channel, reducing the friction of the bed, and obviating the loss of velocity that occurs in an irregular channel.

Mr. Rickman, the editor of the "Life of Telford," thus clearly describes how easily and surely this natural drainage of the Fens was effected, in recounting a visit paid to these works in August, 1830, in company with Mr. Telford:—

"If you cut a canal, you are sure that the removal of earth, and the usual apparatus of locks, will attain your object; but the improvement of harbours, and of drainage by rivers, depends on the management and direction of natural causes and effects; in which, I may say, observation has been so torpid, that till twenty years since, much more harm than good had been the result of interference. All the reports of Mr. Smeaton, and some made scarcely sixteen years since, prove that in large drainages near the sea, natural outlets or rivers were always recommended to be stopped by dams and sluices, to prevent the tide from entering, which obstacles equally prevented the drainage water from free passage outward.

"Three inches fall (downward slope) in a mile makes water move slowly; at four inches' declivity in a mile, water acquires a moderate velocity, sufficient for any drainage operation; so that the sill of a sluice, if laid a yard too high, will prevent the natural drainage of twelve miles above it (three inches to a mile). On the same principle, if a drainage outlet, obstructed by what may be almost deemed the *caprice* of winds and tides, and of accumulated sand-banks in consequence, shall double its length, and creep through a dubious, crooked channel, it is evident that a three or four inch fall may become one inch, which is ineffectual.

"The sound principle which results from these facts is, to give free ingress to the tidal water, guarding against inundation by raising the bank of your river, and also straightening its course, so as to lose no downfall. This increased downfall and increased tidal water is made to bear directly upon the old sand-banks; and if the connexion with deep water can be established in this manner, you obtain a harbour of easy access, and the old-fashioned precarious drainage of land by windmills becomes unnecessary, the dams which previously hindered daily drainage at low-water being for ever removed.

"All this was to be seen in progress below Wisbeach and Long-Sutton-bridge; and the impetuous outfall of the water, in the recess of a spring tide, had forced its way through the sands in the beginning of August, 1830. With a view to this event, the old channel of the river Nene had been boldly dammed across in the middle of July, and the current turned into the straight cut prepared for it. All this constitutes the 'Nene Outfall.' When I saw it meet the sea, four miles below the washway (now the drawbridge at Long-Sutton), at three-quarters ebb, the torrent rushed down four feet in the last quarter of a mile: this, of course, carries off the sand daily; and, by the law of nature, the four-feet fall will recede inland, until nearly a uniform inclination or slope shall penetrate to Wisbeach, which may become a seaport of importance; and above it 180,000 or 200,000 acres of fen-land will retain nothing of its hitherto nature, except unparalleled fertility."—(Life of T. Telford, 1838.)

This additional excavation of the bed of the new outfall, both here and at Eau-brink, by the scouring action of the current, seems to have been part of the original design, and not an unlooked-for advantage; it is only applicable when, as here, the material is carried into the sea, and by the action of the tides removed from the mouth of the channel. The original dimensions of the upper end of the Nene Outfall cut were 410 ft. wide at top at the level of high-water of spring tides, 250 ft. wide at bottom, and 20 ft. deep; giving a transverse area of 6600 sq. ft., more than 1000 sq. ft. greater than that of the Eau-brink. The low-water surface in this outfall was nearly 5 ft. lower than that of the Eau-brink cut, for we have seen that low-water of spring tides was at King's Lynn 5 ft. higher than that in the open in the Wash; but in the Nene Outfall dead low-water has been brought up, all but a few inches, to the inner end of the new channel, a most important difference in the drainage of the lands on each side. As in the Eau-brink, here also the works of the new cut have enclosed and reclaimed the large area of the former channel, more than an equivalent for the quantity used in excavating the new line. The expense of executing these works was, for the Nene Outfall, £200,000; for the North Level drain, £150,000; and the Eau-brink cut, about £200,000; and the ultimate gain, direct and indirect, is estimated at double the cost incurred.

(LIX.) The city of Marseilles is now supplied with water derived from the river Durance, one of the tributaries of the Rhone, having its source in the glaciers on the western slopes of the Alps. The water is taken off at a point near the bridge of Pertuis, 614 ft. above the Mediterranean, distant 28 miles from Marseilles in a direct line; but, from the circuitous course of the artificial channel, that distance is increased to 51 miles. In this length the total fall of the aqueduct is 123.5 ft., which still leaves a height of 490.5 ft. above the sea-level, at a point near the village of St. Antoine, where part of the water is applied to the irrigation of about 20,000 acres of land, on the arid slopes of the hills which nearly surround the city, part being used for mill-power; and part, after passing through settling reservoirs and filter beds, is used for domestic supply, fountains, &c., the distributing reservoirs being about 240 ft. above the sea-level.

The total fall above-mentioned gives an average inclination of 2.437 ft. per mile, or 1 in 2166.6; but this was very unequally distributed, and with great skill, by the engineer, M. Mont-Richer.

Wherever large structures of masonry became necessary, the rate of inclination was increased, in order to effect the discharge of the same quantity of water through a smaller sectional area; and in the tunnels, which were very numerous, as also in rock excavations, the sectional area, and consequently the expense of the works, was reduced, by adopting in such parts of the course inclinations much greater than in the excavations and embankments formed in ordinary earth,—the same cause which rendered such parts of the line expensive to form, namely, the higher cohesion of the material, rendering them also able to resist the action of the increased velocity of the water, and still more so when the sides and bottom consist of masonry. Figs. 85 to 90 inclusive, represent the transverse sections used in these different situations (with the corresponding inclinations written above them), at a scale of 5 ft. to 1 in.

From the statement of the engineer, it appears that the law authorising the construction of the canal permitted only 199.3 cb. ft. per second to be taken from the Durance during the period of low-water, but during all other periods 346.64 cb. ft. per second.

The works are designed to convey 416 cb. ft. per second, at the highest line they admit of. The above numbers, expressed in cb. ft. per minute, give, in round numbers, 12,000, 21,000, and 25,000 cb. ft. per minute.

It will be perceived from the dimensions figured on the transverse section in earthwork, Fig. 85, that the side slopes are inclined at $1\frac{1}{3}$ to 1; and therefore their length will be to the depth as 5 to 3, p. 140; and with the depth of 1.5 mètres, each slope will be 2.5 mètres in length, and the wetted border 8 mètres. The mean width will be 5 mètres, and consequently the area 7.5 mètres square; and the hydraulic mean depth equal to $7.5 \div 8 = 0.9375$ mètres. In English measure it becomes 80.74 sq. ft. $\div$ 26.25 = 3.076 ft.; and from equation (*e*), § 112, we have

$$v = 100\sqrt{\frac{1}{3333} \times 3.076} = 3.04 \text{ ft. per sec.}:$$

this, multiplied into the area, gives $80.74 \times 3.04 = 245.45$ cb. ft. per second, or 14,727 per minute, about 3 per cent. greater than the result of M. Mont-Richer, who calculated by the formula,

$$v = 56.86\sqrt{\frac{1}{3333} \times 0.9375 - 0.072}, \text{ all in mètres.}$$

With the depth of 1 mètre, we have the border equal 6.33 mètres, and the area 4.33 square mètres, which gives 0.684 mètres for the hydraulic mean depth.

In the open cutting in rock, Fig. 86, it is evident from the figured dimensions that the inclination of the side slopes is $\frac{1}{10}$th to 1; hence at the depth of 1.5 mètres the length of the side slope will be $\sqrt{1.5^2 + 0.15^2} = 1.5075$, so that the border will be equal to $5.52 + 2 \times 1.5075 = 8.535$ mètres. And the bottom width being 5.52, the top water line at 1.5 depth becomes 5.82, half the sum of which widths, multiplied into 1.5, gives 8.505 square mètres for the area, or 91.55 sq. ft., and $8.505 \div 8.535 = 0.9965$, the hydraulic mean depth in mètres = 3.2695 ft.; hence

$$v = 100\sqrt{\frac{1}{3333} \times 3.2695} = 100 \times 0.0313 = 3.13 \text{ ft. per sec.},$$

and thus the discharge equal to $3.13 \times 90.55 = 283.4$ cb. ft. per second, or 17005 cb. ft. per minute.

The fall over the bridge aqueduct has been made as great as 1 in 250; and with a channel whose transverse section, at 1.5 mètre in depth, is equal to 3.225 sq. mètres, and the wetted border 5.015 mètres, giving an hydraulic mean depth equal to 0.643, that is 2.1097 ft.; hence the mean velocity is

$$100\sqrt{\frac{1}{250} \times 2.1097} = 9.186 \text{ ft. per second,}$$

which, multiplied into the area 34.72 sq. ft., gives 318.94 cb. ft. per second, or 19136 cb. ft. per minute.

This remarkable structure has a total length of 1290 ft.; the extreme height is 265 ft. 9 in.; the average being 174 ft. The surface exposed to view in the elevation is 224,200 sq. ft. The total cost, after the sale of materials used in the work, was £148,000, or £558 10*s*. per foot run, or 13.2 shillings per square foot of elevation; the pressure on the base of the foundations amounts to 210 lbs. per sq. inch, or 13.5 tons per sq. ft.

Like the celebrated Pont du Gard, this aqueduct consists of three tiers of arches; and though this work is that which has chiefly drawn attention to the canal, it seems doubted by competent judges whether it was necessary to carry the bridge aqueduct to so great a height; and, even if brought up to 266 ft. at the highest, whether a single series of arches, so frequently adopted in railway viaducts, would not have been more economical, and sufficiently secure, inasmuch as the conveyance of uniformly-flowing water causes none of that vibration incidental to the passage of rapid railway trains.

On the Croton water-works, which supply New York, the water is carried over the Haarlem river, a navigable arm of the sea, in pipes, supported on a stone aqueduct. The height of the soffit of the arches is not more than that demanded by the authorities having charge of the navigation: this height is below the line of the uniform slope of the bottom of the culvert produced across the river. Accordingly, a cast-iron pipe receives the water at the upper end, and, bending down to the level of the masonry, rises on the opposite side, delivering the water at a height 4 ft. lower than that at which it was received.

By this design the masonry is greatly reduced, with a loss of head in the channel of only 4 ft. It was evidently impossible in this peculiar locality to have carried the cast-iron pipe down to the bottom of the valley, and up the opposite side, as has been done in so many modern works, as, for instance, at the recently opened water-works from Loch Katrine to Glasgow, on which several valleys are crossed by a cast-iron pipe 4 ft. in diameter, the difference of level between the two extremities being such as to give an inclination of 1 in 1000.

In the section of the tunnel lined with masonry, Fig. 90, the slopes of the sides are evidently, as in Fig. 86, at $\frac{1}{10}$th to 1; and therefore, for a depth 1.50 mètres, the length of each is 1.5075, also the length of the arc of the invert will be 3.0244 mètres; for, as the half chord is 1.5, and the versed sine 0.15, the radius will be 7.575 mètres; that is $(1.5^2+0.15^2) \div (2 \times 0.15)$, and the circumference $= 2 \times 7.575 \times 22 \div 7 = 47.614$, which will be to the length of the arc of the invert as 360° is to the degrees, &c., in the invert: this last is obtained from the nat. sin. of half the invert, which is $1.5 \div 7.575 = 0.198$, answering to 11° 26′, so that the arc of the invert contains 22° 52′. Thus the total border will be 6.04 mètres.

In like manner, the area of the segment is equal to the difference between the sector of that number of degrees, radius being 7.575 mètres, and the isosceles triangle whose base is the chord 3, and the sides 7.575; the area of this triangle is, therefore, $7.56 \times 1.5 = 11.34$ square mètres. Now the area of the sector of 22° 52′, and radius 7.575, is 11.455 square mètres, for the area of the whole circle is $(2 \times 7.575)^2 \times 0.7854 = 180.339$, and the sector is the same part of this that 22° 52′ is of 360°, or as 1372′ is to 21600′. The area of the part included between the chord and arc of the invert is therefore $11.455 - 11.34 = 0.115$ sq. mètres, which must be added to that of the trapezium between the water-line and the chord-line, which has for its mean width $3.3 + 3 \div 2 = 3.15$, and the depth being 1.5, gives the area of it 4.725 sq. mètres, so that the whole transverse area is $4.725 + 0.115 = 4.84$ sq. mètres, or 52.1 sq. ft. The border being as above 6.0394, we have the hydraulic mean depth $4.84 \div 6.0394 = 0.8014 = 2.6294$ ft. Hence the mean velocity equals

$$100\sqrt{\frac{1}{1000} \times 2.6294} = 5.128 \text{ ft. per second;}$$

this, multiplied into the area, gives 267.2 cb. ft. per second, or 16032 cb. ft. per minute.

The tunnel, not lined with masonry, and of a fall, as above, of 1 in 1000, has, from the dimensions figured, slopes of very nearly $\frac{1}{10}$th to 1, so that, for the depth of 1.50 mètres, the length of the sides will be 1.5075, and the border $4 + 2 \times 1.5075 = 7.015$, and the area 6.225 sq. mètres; hence the hydraulic mean depth is 0.8874 mètres, or 2.9 ft., and the mean velocity

$$100\sqrt{\frac{1}{1000} \times 2.9} = 5.4 \text{ ft. per second,}$$

which, multiplied into the area 65.77 sq. ft., gives 355.158 cb. ft. per second, or 21309.5 cb. ft. per minute.

Though, at a depth of 1.5 mètre in each different channel, the volumes are thus different in amount; yet when Figs. 85, 86, 87, are running 2 mètres deep, Figs. 88, 90, at 2.10 mètres, and Fig. 89 at 1.80 mètres in depth, that is, about the greatest each respectively admits of, the discharge will be in all very nearly equal, and about 25,000 cb. ft. per minute—the greatest volume permitted by law being 21,000.

In the discussion upon the paper giving an account of the works in this example, Mr. Simpson, Engineer-in-chief of the Bristol Water-works, gave a description of the construction by which he carried the supply of water for that city across several valleys which intersected the line,—a wrought-iron oval tube, of the form and dimensions shown in Fig. 95, forming at once the channel of conveyance and the beam supporting the weight between the piers. Speaking of one of these, he says—

"The aqueduct was continued across the Coombe, or ravine, by means of an oval wrought-iron tube, 354 ft. in length, supported at intervals of 50 ft. on piers of masonry, nearly 60 ft. in height. The internal dimensions of the tube accorded with those of the stone aqueduct; and the ends were connected with it by means of stone tanks, and collars of clay puddling, 10 ft. in thickness, in which the ends were embedded, so as to allow for any expansion, or contraction, and to prevent leakage. The tube rested on cast-iron saddles, fixed to the piers and abutments; and provision was made in the saddles, by means of friction balls, to allow of the tube expanding and contracting with the variations of temperature, each way from the centre saddle, which rested on a pier of equilibrium. The balls of the centre saddle admitted of slight motion laterally, but prevented the tubes moving end-ways in either direction.

The following were the dimensions of the tube, of precisely similar construction, across the valley of Leigh Down :—

Length of wrought-iron tube, ...	834 ft.
Length of aqueduct between abutments,	800 ft.
Span from pier to pier, ...	50 ft.
Greatest height, ...	40 ft.

Total cross sectional area of oval tube, 13 sq. ft.; water area, 10 sq. ft.; fall, 1 in 1056, or 5 ft. per mile; capability of the aqueduct for carrying water (10 sq. ft. transverse area), 2200 cb. ft. per minute, or 20,000,000 gallons per day; and therefore the wetted border is 7.44 ft., and hydraulic mean depth, 1.344 ft. Weight of iron tube, 84 tons; weight of cast-iron saddles, &c., 26 tons; weight of water, 230 tons: total weight on piers, 340 tons—giving a pressure on the masonry of about 1¼ tons per sq. ft.

Cost of tubes, saddles, bolts, &c., fixed,	£2617
Cost of masonry, &c., ...	2000
Total cost of aqueduct, . . .	£4617

Cost per lineal foot, £5 10*s*.

The wrought-iron plates of which the tubes were composed were $\frac{5}{8}$ths of an inch thick at the bottom, and $\frac{7}{16}$ths of an inch thick at the top. The section was oval, the transverse diameter being 3 ft. 9 in., and the conjugate diameter 4 ft. 7 in.

Very light and efficient aqueducts were thus formed, spanning these wide valleys at a cost not exceeding £14 to £16 per lineal yard, including all expenses.

The aqueduct and tubes were of sufficient capacity to convey a larger quantity of water than the cast-iron pipes of 30 in. diameter, which were laid between these valleys and the city; the dimensions of the former having been in some degree decided by the minimum area in which a man could conveniently work.

In forming a tube of wrought-iron plates, which are always joined by riveting, it is necessary that a man, called a holder-up, remain on the inside, while the rivetters on the outside by hammering form the head upon the rivet: it is with reference to this, that the remark mentioned above, as to the internal dimensions of the tube, was made. Each line of cast-iron pipes was calculated to convey 8,000,000 of gallons of water per diem, and the land purchased for the pipe-track was of sufficient width to admit of another line of pipes at any future time.

It was stated that the large fall of 5 ft. per mile for the aqueduct in masonry, and in the wrought-iron tubes, and that of 10 ft. per mile for the cast-iron pipes, 30 in. in diameter, was adopted with a view to meet, by a high velocity, the danger of incrustation; the water being derived from the limestone district of the Mendip Hills. The result has been perfectly satisfactory in this respect.

It may be confidently stated that no English engineer would adopt, with their command of, and experience in, cast and wrought-iron, a design such as that at the Aqueduct of Roquefavour, and would probably have crossed that valley at one-third of the expense incurred. If the line of the water-surface crossed a valley at an extreme elevation, a metal pipe would have been bent downwards on one side to some convenient depth, led across the valley, and again upwards, leaving sufficient fall between the entrance and delivery of the pipe to insure a sufficient discharge, as practised on the Glasgow and also the New York Water-works.

(LX.) The transverse sections, Figs. 91 to 94, represent the forms of the channels used by the Commissioners of Drainage in Ireland in their Main Drainage works, the object of which was the lowering of the beds, so as to keep down flood-water line in rivers, and straightening and improving their courses, so as to give a more ready discharge for the waters of unusual rain-fall. Many considerations arise in any attempt to improve the course of a river, independently of the property question, which is sure to occur; for the centre line of streams being very frequently the boundary, not only between counties, parishes, or other denominations, but also between private estates, the rights of corporations and individuals are unavoidably interfered with in rectifying a circuitous channel, or lowering a mill-weir: these matters must either be arranged by a general or special Act of Parliament, or by an agreement among all the interested parties. Not only these intricate questions arise, but what may be deemed the physical characteristics of the river demand a close attention.

Every river has a special and well-defined territory, which it drains; the boundaries of it, or its water-shed line, are not always marked on maps, unless they happen to coincide with some territorial division; nevertheless, they are as well-defined upon the surface of the earth as the lines of the water-courses themselves. In fact, upon the irregular surface of the earth we may recognise three descriptive lines—the contour, or horizontal line; the line of the water-courses, that is, the valley line or thalweg; and the line of the water-sheds: these last two having the property of being always perpendicular to the former, and being, one the steepest possible, the other the least inclined that can be drawn with that condition.

If, then, measurements of the rain-fall be taken by the rain-gauge, extending over many years, so as to obtain a true average annual fall, we may calculate the total volume descending on the particular territory, or catchment-basin as it has been called, of the river.

It will be necessary, in the next place, to estimate what portion of it is lost by evaporation, &c.; this depends on the character of the surface, as to its slope, and as to its material constituents. The more rapidly the rain-fall is delivered off the surface into the channels, and the more rapidly they discharge it, the less is lost by evaporation. A bare and rocky surface acts like the roof of a house, or a district consisting of a closely built city, the paved streets of which, and the roofs of its houses, discharge suddenly almost all its rain-fall, leaving very little to be evaporated; on the other hand, a surface of spongy peat, porous chalk, or cultivated land, retains for a much longer time the moisture it receives, and therefore gives off a large proportion of it by evaporation. In addition to the character of the surface, as more or less retentive, the inclination of the bed of the stream has also a marked effect in this respect: an inconsiderable rate of fall, and the occurrence of lakes along the course, are greatly favourable to evaporation, and consequently modify very materially the volume of water to be conveyed down the water-course.

Many circumstances concur to render Ireland peculiarly adapted as a field for the beneficial exercise of this branch of civil engineering. If we look to the map of Europe, we perceive Ireland stretching westward into the Atlantic Ocean beyond all other lands; and from its insular position, and exposure to the prevailing winds from south and west, which come in from the ocean charged with moisture, we find the average rain-fall very great. It may be estimated at 36 in. for the whole island, and is probably 40 on the western coast, of which it is supposed that about $\frac{2}{3}$rds are lost by evaporation, and only $\frac{1}{3}$rd flows down the rivers.

And, secondly, though the geological structure of Ireland is similar to that of England, yet the relative geographical position of the different rocks is essentially different. In England, the mountain ranges, consisting of the primary and transition classes, are situated near the west coast, and the newer strata are successively accumulated on each other to the east and south, in which directions the country is comparatively flat. In Ireland, on the contrary, the coast is for the most part mountainous, while the interior is flat, and rarely presents hills of considerable elevation. Thus we find the primary mountains of Antrim, Derry, and Donegal, occupying the north and north-west coasts; those of Sligo, Mayo, Galway, and Kerry, the west and south-west. The slate districts of Cork and Waterford form the south and south-west; while the mountains of Wicklow, Louth, and Down, are situated on the east coast.

These mountain tracts rarely extend more than 20 miles inland; and we find the interior, with trifling exceptions, composed of flat or gently-swelling plains of limestone, covered by a rich and fruitful soil. This peculiar conformation of the surface has been the origin of the great number of rivers with which the coasts abound; they have their sources in the neighbouring mountains, whence they flow, in short but rapid courses, directly to the sea. The rivers of the interior, on the other hand, flow but slowly through the flat limestone districts which occupy two-thirds, at least, of the surface of the island. And although summits intervene between the valleys of the several rivers, still these summits are so low, and the ascents so gentle, that to an inexperienced eye the whole country appears to consist of one uninterrupted plain.

The Grand and Royal Canals each traverse the island from east to west, across part of the limestone plain; the summit of the former is only 270 ft., and the latter 310 ft. above the mean sea-level. The general average height of this flat limestone plain may be set down at about 350 ft. above the sea, though wide areas are found, which do not rise above 250 ft.

The Shannon forms the most striking example of the rivers of the interior flowing slowly through the flat country. This river has its source in a limestone cavern, situated in the valley of Lough Allen; it issues from a circular gulf, about 50 ft. in diameter, and at once assumes the character of a considerable stream; the lake itself is generally considered to be the head of the Shannon, though 5 miles south of this source. Lough Allen is elevated 160 ft. above the level of the sea, while the course of the river to the tide-way at Limerick is 140 miles in length, which gives an average fall of but 1 ft. 2 in. per mile; but this average, small as it may appear, is not equally distributed, as the greater portion of it, amounting to 98 ft., takes place between Lough Derg and Limerick, near Castleconnell, a distance of but 15 miles, leaving a fall of only 62 ft. between Lough Allen and Lough Derg, in a distance of 127 miles, which is rather less than 6 in. per mile; and, further, part of this distance, between Tarmonbarry and Banagher, nearly 40 miles, falls but 8 ft., or 2.4 in. per mile.

And not only is the longitudinal inclination of the bed thus exceptionally low, but, transversely, the country on each side is nearly level; and the tributaries of this river, traversing these wide flats, have also very sluggish courses, and partake of the character of the main stream. Extensive works have recently been completed on the Shannon, intended to render it navigable: and, partly with a view to reduce the evil of inundations, the crests of the weirs have been greatly lengthened, and lowered, and the bed of the channel dredged. The latter object has not, however, been attained; and it may be presumed that the inundations to which the bordering lands are still liable may be, in a great degree, referred to the effect of the main drainage of its tributaries, and their lesser branches, which have, subsequently to the Navigation Works, been improved, and thus, as will be mentioned below, send down in a shorter space of time the rain-fall of the upper districts, giving to the main stream a volume of water which even the increased discharging powers of the navigation cannot cope with : the floods may last a shorter time, but they rise as high as formerly, and are equally injurious. It is difficult to imagine a river-course more favourable to evaporation than that of the Shannon, expanding as it does for a large part of its length into wide lakes, and having on each bank extensive flats, consisting either of clays of retentive character, or extensive bogs. This evaporation lowers the temperature of the atmosphere and of the land alike, and one important benefit resulting from the general drainage of such districts is the counteracting of this disadvantage.

The other rivers of the interior, including the large tributaries of the Shannon itself, though generally having a greater inclination than it, must yet be considered as very flat, and requiring the regulation and improvement of art.

Thus, from the great amount of rain-fall, and the level character of the surface of the country combined, the rivers of the interior of Ireland derive their tendency to rise above their banks, and inundate the low and level lands on either side.

The engineer, whose duty it is to benefit the lands on each bank of a river, by regulating its highest flood-water line to a certain depth below the surface of the district through which it flows, finds the estimate of the maximum rain-fall in some short period, as 24 hours, of higher importance than the determination of the true annual mean: this last is more essential when great storage and compensation reservoirs are about being formed for the supply of cities, and the compensation of the mills for the quantity thus drawn off. In many places the greatest recorded floods have been observed to take place when, after a deep fall of snow, a warm southerly wind brings a heavy continuous rain; the snow already covering the ground is thus melted, and flows off with the rain, in effect as if both had fallen simultaneously.

Notwithstanding the ever-varying quantity brought down from day to day, we may consider that an equilibrium has, in the lapse of time, been formed between the erosive action of the descending waters and the resistance of the soil through which it flows; thus a light or alluvial soil will be found traversed in a tortuous and winding course, effectively diminishing the rate of inclination per mile by lengthening the course, until, in all the usual volumes carried down, the bed may be said to remain unaltered; and were the annual rain-fall delivered off at a uniform rate, this equilibrium would be undisturbed; but a sudden flood, by its swift and impetuous motion, cuts into the banks and bed, effecting at times a lasting change in the course on its subsidence.

Hence it is that, in attempting to improve the courses of rivers, such caution is required; for if this balance between the cohesion of the soil of the banks on the one hand, and the erosive action of the flowing water on the other, be disturbed to any extent beyond very narrow limits, we can look for no permanency in the works after their completion. And again, if, as is frequently the case, the upper districts of a river, or some of its branch streams be straightened and lowered, the flood-waters from such portions are delivered down much more suddenly than formerly, to the great injury of the lands and mills on the lower part of the course. It has also occurred that one tributary of a river, entering its course at a certain point, discharges all the flood-waters of a heavy rain-fall before the rise in the main stream, from the same cause, has descended so far. Now, the rectification of the upper part of the river may make its rise at the point of junction coincident with that of the tributary, to the injury of the lower reaches, and danger of the bridges and mills. Indeed, the field or thorough drainage of the country, independently of any interference with the river channels, has, when very generally carried out in the catchment basin, been observed to alter in a material degree the character of the streams proceeding from it. The floods are much more rapid and impetuous, but last for a much shorter time; and in periods of dry weather, on the other hand, the quantity flowing down at low-water is diminished. It is probable that on the whole the volume discharged is larger, after such improvements, than it was formerly, since less is undoubtedly carried off by evaporation.

In the "Life of Telford," an interesting confirmation of the above remark occurs. One of his earliest works was the rebuilding the bridge over the Severn, at Buildwas, about 10 miles below Shrewsbury.

The old bridge, which was probably 500 years in existence, consisted of narrow arches, which were a great obstruction to the navigation, and it was destroyed by an uncommonly high flood in the year 1795. The Severn bridges were peculiarly liable to this danger; for, while the low lands at the foot of the Welsh mountains remained unenclosed, the floods speedily covered the flat ground, not deeply, but to a great extent of surface, and drained off gradually; but after these lands were embanked, and the floods were thus confined to the natural channel, they rose in it to a greater height than formerly, and passed off with greater velocity; thus the bridges were endangered, and the navigation injured, there being too much water at one season, and at another not enough.

Subsequently, when engaged on the Caledonian Canal, he was led to contrast the character of the Ness, which supplies the summit level, with that of the Severn. The former has on its course several considerable lakes, which, acting as reservoirs, receive the sudden floods of a mountainous district; and their surfaces, gradually rising, as gradually subside after the cessation of the rain; the important element of a lengthened time of discharge being introduced through the agency of the extended area of the lakes.

Sir Humphrey Davy, many years since, in a work on fishing, named "Salmonia," remarked on the alteration that the under-drainage of the farms had produced in the streams in Scotland in which he followed this amusement; the rain being all delivered off very rapidly, and in fine weather the beds of the water-courses becoming nearly dry. In parts of France and Italy, the cutting down of forests on the slopes of the mountains has altered the character of the rivers flowing from them, rendering more frequent and destructive the inundations of the plains through which they flow.

Thus we readily perceive that the progress of cultivation and agricultural improvement tends to produce injurious alteration in the streams flowing from such districts, and calling for the aid of art to remove or mitigate their effects. And it also appears that the work undertaken should be universal over the whole district of a river, and not confined to one or a few of the tributaries, or to its upper districts only; for then we should but increase the first-named evil, and deliver off still more rapidly that volume of water which the improvement of the lands had already sent down, in a very much diminished space of time.

In general, the longitudinal section of a river, from its source to its termination, has a much greater inclination in the upper reaches; and, gradually decreasing in the rate of fall, as its volume is augmented, presents a curve convex to the horizontal line drawn through the lowest point. If this section be taken along the centre line of the course, we find that the bottom of the bed, though conforming to this curve in general, is yet a very irregular line; the sections Figs. 79 and 84 will give some idea of it. It must be remarked, however, that the sections of the beds of rivers, being more exaggerated in the vertical scale than other engineering profiles, give, perhaps, a too great idea of the unevenness. The study of the plan and transverse sections of a stream will best explain this fact, so unfavourable to the discharging power of the channel. Now, it is always found that in the bends of rivers the concave side, as at A, A′, Fig. 96, is deepened by the action of the current eroding both the bed and the bank, as in the transverse section; the same thing occurs at the succeeding curve, as at C, C′; and in the intermediate part, as B, B′, the water flows in a bed symmetrical on each side of the centre line, more in the manner shown in the transverse section with the same letters. The longitudinal section on the centre line, Fig. 97, results from these well-established facts,—being shallower at B, B′, and deeper at the curves,—though the section on the central line does not give the deepest portion, which is found still nearer the concave bank.

It is well known to military engineers that the best direction for attempting to ford a river is in a line as *a*, *a′*, that is, on the shallow bank between opposite sinuosities, and in this part also are found the greatest obstacles to the navigation of the stream.

Thus the longitudinal section, Fig. 97, exhibits, not a uniform curved line for the descending bed, but frequent depressions and elevations succeeding each other; the causes of this result of moving water must be sought in the absence of a homogeneous structure in the clays or gravels through which the stream flows, and in the continual alternation of the volume passing down between the limits of great floods and lowest water.

The following Table gives the ratio between the lowest and highest states of the rivers named. It must be remarked that some uncertainty necessarily attends the estimate of the latter quantity:—

British.	Place of Observation.	Cb. ft. at low-water.	Cb. ft. in high floods.	Ratio.
Shannon, . .	Killaloe, . . .	100,000	1,000,000	10
Tyne, . . .	Ryton Island, .	16,025	1,923,000	120
Tay, ...	Perth, . . .	20,000	660,000	33
French.				
Allier, . . .	Le Guétin, . .	565	211,920	375
Garonne, . .	Toulouse, . .	1271.5	201,324	158
Loire, . . .	Briare, . . .	1130	353,200	312
Rhine, . . .	Kehl, ...	13,422.6	166,004	12
Saône, . . .	St. Jean de Losne,	1032	28,256	29

The Nile and the Ganges are estimated to have a ratio of 5 to 1 between their highest discharge and ordinary flow.

Mr. Jessop, an eminent canal engineer of the beginning of this century, in reporting on the supply of water to be obtained from the natural streams at the two summits of the Grand Junction Canal, where it crosses the two ridges of hills in the secondary formations, north of London, gives some curious practical remarks with reference to the variable quantity of water to be found in the streams flowing from different districts, according to the season:—"In a clay soil, such as at Braunston, the extremes of wet and dry seasons differ as much as one thousand to one; in chalky and gravelly soils, the difference is seldom more than as four to one; for in the former case heavy rains produce great floods, and little is absorbed; in the latter case, there are seldom any floods, for almost the whole are absorbed; and it only operates for a while to increase the supply by the springs, which may be considered as the discharges of natural reservoirs, dispensing frugally for many months what would be lavished from the surface of clayey soil in a few days."

In fact that one-third of the total rain-fall, which is computed to flow down the water-courses, divides into two portions: one passes immediately from the surface of the ground to the valley lines and rivers; the other, percolating through the soil, at length reaches the same destination, but through the intermediate action of springs, forming the least variable part of the total volume, and in long-continued absence of rain-fall almost the only supply.

Our first surmise would naturally be that a water-course would exhibit a right line, in plan, descending down the steepest possible line, which would be the slope of the valley in which it was flowing; but a very slight cause is sufficient to deflect the current, and, once commenced, we find the deflections are reproduced successively on the opposite banks, like the letter S; and the concave sides,—being more constantly acted upon in the lower part than in the upper, because great floods, which fill the course, and reach the higher part of the banks, occur but seldom, and are short in duration, compared with ordinary floods and low-water,—are undermined, and thus continually fall in; the erosive action recommencing as soon as the *debris* have been carried down the stream; and thus the sinuosity continually increases, until it meet with material capable of resisting its further progress, and the average velocity of the stream becomes nearly in equilibrium with the soil that the bed passes through.

The object of the engineer in the work of improvement must therefore be to give, in the first place, a section so proportioned to the maximum quantity to be discharged, and rate of inclination, as that the surface of the water must not rise, even at that time, so high as to injure the under-drainage of the bordering lands, dependent on the improved channel for an out-fall. It is usual to assign 3 or 4 ft. below the land surface for this highest level of winter floods. If the openings of the pipe-drains are covered in floods, they are very liable to choke by the sediment from the river water, deposited in them from its comparatively quiescent state. This transverse section must increase with the increased volume of the stream in its descent, regard being also had to its greater or less rate of fall. In plan, the sinuosities must be very greatly reduced, or they may be replaced by a rectilineal course, when such is possible. The velocity must also be kept within that which, in each particular locality, according to the nature of the soil, would destroy the bed or the side slopes; and, as the rate of fall is not capable of any very considerable alteration, it is the modification of the form of the transverse section which must be relied on to regulate it, a greater divergence from the best discharging form diminishing the velocity proportionally, though in this also we shall find the choice much restricted.

Fig. 91 represents the transverse section of a channel in which the line of high-water of winter floods is supposed to be kept 4 ft. below the surface of the lands on each side. The ratio of the depth of the water to the mean width, i. e. 4 ft. to 30 ft., or 1 to $7\frac{1}{2}$, is not widely different from that which would be deduced from observations on natural water-courses at the best parts of their channels. With a fall of 3 ft. per mile, the velocity of flood srunning 4 ft. deep will be 3.97 ft. per second; for with this depth, mean width, and the side slopes $1\frac{1}{2}$ to 1, the area is 120 sq. ft., and the border $24 + 2 \times 4\sqrt{1.5^2 + 1} = 38.42$ ft., and the hydraulic mean depth $120 \div 38.42 = 3.1234$, consequently the velocity (j) p. 134, is $0.92\sqrt{2 \times 3 \times 3.1234} = 3.97$ ft. The discharge is, therefore, $3.97 \times 120 = 476.4$ cb. ft. per second.

If the velocity of 3.97 ft. per second (nearly 4 ft.) be considered either too great, or the contrary, for the tenacity of the ground through which the channel is excavated, and the true limit be known either by experiment or observation, we can proceed to calculate the dimensions of the channel that, with a given fall and side slopes, will convey this volume of water at the assigned velocity.

Thus, supposing 4 ft. per second was too great, and that it was proved that 3 ft. was the limit which the soil could bear, then, as we know the discharge, namely, 476.4 cb. ft., and this velocity, we have the transverse area, which is always $Q \div v$, $476.4 \div 3 = 158.8$ sq. ft.; hence, if we put c for the wetted border, we can obtain it, the fall being 3 ft. per mile as before, from the equation $0.92 \sqrt{6 \times \frac{158.8}{c}} = 3$, and, therefore,

$$c = 6 \times 158.8 \div \left(\frac{3}{0.92}\right) = 89.89.$$

If now we put z for the depth, and the slopes be as before, $1\frac{1}{2}$ to 1, we have

$$z^2 \times 1.5 + z(c - 2\sqrt{n^2 + 1}.z) = 158.8.$$

or $-2.16z^2 + zc = 158.8$, changing signs and dividing

$$z^2 - 42.8z = -75.6, \text{ and the depth } z = 1.85.$$

The bottom width is, therefore, 83.19 ft., and the top-water line 88.74 ft. Such a channel would be expensive, from the quantity of land required, which would be 100.6 ft. in width, the flood-water surface being kept 4 ft. below the level of the land on each bank.

The coefficient 0.12′12′, multiplied into the width in feet, gives the acres per mile required for any line of work of uniform or average width. Approximately, one-tenth of an acre for each foot in width, with one-fifth of this added, will give a near result. Thus 100.6 × 0.12′12′ = 12.2 acres per mile. It is probable that, to avoid such a transverse section, the side slopes would, if stone were readily to be obtained, be built nearly vertical, and the bottom artificially protected from scouring, so that the former velocity, or even a greater, could be maintained.

On the other hand, if the soil was such as to admit of 5 ft. per second, instead of 3.97 ft., we would find that the data, as to volume, side slopes, inclination per mile, and velocity, may be inconsistent; for these data necessarily determine both the transverse area, and the border also. Now these two last are so connected, that the former has a minimum border, and the latter a maximum area, possible; and, if either of them be assigned, we perceive at once the limits for the other.

The volume of water to be conveyed, namely, 476.4 cb. ft., divided by the assigned velocity, 5 ft., gives the area of the trapezium requisite 95.48 sq. ft. And $0.92 \sqrt{2 \times 3 \frac{95.48}{c}} = 5$ ft., gives $c = 19.8$ ft. for the wetted border.

From the Table, p. 138, we find the area of the trapezium of maximum discharge, side slopes $1\frac{1}{2}$ to 1, to be $p^2 \times 2.106$; if we put this equal to 95.48, we have $p = 6.733$; and half this, that is the hydraulic mean depth, into the border, gives the area; so that 3.3665 divided into 95.48 gives the least possible border, or 28.37. Also, as the border is to the bottom width as 7 : 1, *q. p.*, we have, with a border equal 19.4, the maximum area 44.8 sq. ft.

One of the inconveniences to be carefully guarded against, in forming new channels, is of a character directly the reverse of the injury sustained by erosion, that has been spoken of above, and that is the deposition of material carried down either in floods, or in the ordinary and average state. Hence it has been a great object, when the ground will admit of its adoption, to design channels which, under the given circumstances of the case, may have a permanent form of bed and transverse section, subject neither to erosion from a velocity too great, nor deposition from the contrary.

In Fig 92 is shown a general drawing of a modified form of transverse section, which may be executed according to detailed working plans. It is excavated 2.22 ft. lower than that in Fig. 91; and, although the same as it is in area, namely, 120 sq. ft. (to flood line), has a greater velocity, and consequently greater discharge also. It is evidently a nearer approach to the best form of discharge than Fig. 91, and may be considered an application of the principle, "cut deep," adopted in the later works.

The depth being 6.22, and the bottom width 10 ft., we have the border $10 + 2\sqrt{n^2 + 1} \times 6.22 = 32.39$ ft., and the area $6.22 \times 10 + 1.5 \times 6.22^2 = 120$ sq, ft.; and $120 \div 32.39 = 3.70$, the hydraulic mean depth; this multiplied by twice the fall of 3 ft. per mile gives 22.20, and the velocity is equal to $0.92 \sqrt{22.20} = 4.71 \times 0.92 = 4.33$ ft. per second; and therefore the discharge $4.33 \times 120 = 519.6$ cb. ft. per second; comparing this with that of Fig. 91, namely, 476.4, we have an excess of 43.2 cb. ft. in favour of Fig. 92.

We can from the Table, p. 138, readily determine the best trapezium of 120 sq. ft. area, and side slopes $1\frac{1}{2}$ to 1, for $p^2 \times 2.106 = 120$, and thus the depth of the water, $p = 7.55$ ft., and the border 31.8 ft.; and as the border is to the bottom width in that figure very nearly as 7 : 1, we have 7 : 1 : : 31.8 : 4.54 ft. for the bottom width; the top width is consequently 27.19 ft., and the land required 39.19 ft. wide, or 4.75 acres per mile.

Now, as the soil wherein this figure is used is sufficiently tenacious to resist a velocity of 4.33 ft. per sec., advantage has, with good judgment, been taken of this to save land and excavation, by giving a steeper slope to the sides, above flood-water line, namely ½ to 1, which adds but 4 ft. to the width of the top water-line, making the land required for Fig. 92 to be 32.66 ft. wide, and 32.66 × 0.1212, gives 3.96 acres per mile.

The width at surface of land for Fig. 91 is 48 ft., and the land required 48 × 0.12′12′ = 5.82 acres, giving a difference of 1.86 acres, or 1 acre 3 roods 10 perches per mile in favour of Fig. 92.

Comparing the excavation per mile required for each channel, the first has 32 sq. yards, the second 26.96 in the transverse area to surface of land; the difference, 5.04 multiplied into 1760, gives 8870.4 cb. yards per mile in favour of Fig. 92.

A variation in the form of this last is shown in Fig. 93, which may be adopted when the lower part of the excavation is sandy, or otherwise bad soil. A slope of 1 to 1 is carried down from flood-water line about 4 ft. (according to circumstances), and another of about 3 to 1 for the remaining depth of 2.22.

The bottom width is 7.34 ft., for the top water-line being, as in Fig. 92, equal to 28.66 ft., the sides at 1 to 1 reduce this, on 4 ft. depth, to 20.66 ft.; and 3 to 1 for 2.22 ft., reduces that again to 7.34 ft., as above.

Thus the area for the part at 1 to 1 is 98.64 sq. ft.; for that at 3 to 1 is 31.08, the sum being 129.72 sq. ft., and the border being 32.68 ft.; the hydraulic mean depth is therefore 3.97 ft., and the velocity equal to $0.92\sqrt{2 \times 3 \times 3.97} = 4.49$ ft. per second, and the discharge $4.49 \times 129.72 = 582.44$ cb. ft., or 62.8 cb. ft. greater than Fig. 92.

The land is the same for both; the excavation per mile for Fig. 93 is, however, 1900.8 cb. yards per mile greater than Fig. 92, the transverse section of the former being 28.04, the latter 26.96 sq. yards; the difference 1.08 into 1760 giving the quantity above mentioned.

Fig. 94 shows another modification of the transverse section, which may be used to suit soils of varied character, by which a still further saving of land may be effected. From a base of 10 ft. we have side slopes of 2 to 1 on each side, for 2.22 ft. vertical; for the next 4 ft. side slopes of 1 to 1; and the upper 4 ft. above top water-line ½ to 1. Thus the area of the lowest portion is 32.06 sq. ft., and of the upper 91.52 sq.

ft., together 123.6; and the border being 31.24, we have the hydraulic mean depth 3.956, and the velocity, calculated as before, 4.48 ft. per second, the discharge consequently 553.7 cb. ft. per second. The breadth of land at surface will be 30.88 ft., requiring 3.74 acres per mile.

Fig. 98 is a tranverse section, taken from a lithographed circular of the 7th October, 1847, issued from the office of the Commissioners, on which the old channel is given in dotted lines. The base of the new water-course is an arc of a circle, whose radius is 1½ times the depth of the channel; the portions of the old channel on each side are carefully filled in, and, being nearly altogether above the new flood-water line, are not liable to injury from the action of the stream.

The excavated material in all these works was placed in a spoil bank on each side, the nearest point of it being 2 or 3 yards from the edge, due care being taken that the small lateral drainage should not be interrupted.

Although none of these figures are trapeziums of exactly the best discharging form, with their given side slopes, yet they, perhaps, are as near an approximation as would be safe to adopt in practice; the depth would, we have seen, § 115, be otherwise too great in proportion to the width. Fig. 99 gives a trapezium of best form, with compound slopes and circular base, the hydraulic mean depth being half the depth of the water, as mentioned in p. 141.

Many of the rivers, undertaken by the Commissioners, have been finished without any excess above the estimate, and have benefitted the districts through which they flow in a very great degree. But, unfortunately, it was otherwise in the greater number of cases; and a few instances of successful resistance in the courts of law, by the proprietors, to the payment of the assessment above the original estimate, brought the Commission to a close,—Parliament finally voting one million and a half to clear off these extra charges; many important districts in Ireland being left unimproved.

It seems impossible, by any system of public works, whether drainage, roads, or railways, to fulfil the duty attempted to be thrown upon the Commissioners of Public Works in Ireland, subsequent to 1846, namely, to give relief to a starving population of unskilled labourers. Such works require a long preparation of surveys and sections, and letting of contracts; and, even if in full operation, the number of skilled workmen, compared with unskilled, is very large. In drainage works, bridges have to be taken down, and others erected; temporary timber bridges being put up to keep open the roadway, or the existing structures under-pinned; weirs have to be removed, or lowered and lengthened; and the mill-wheels lowered in proportion, with corresponding alterations in the machinery and sluices.

If contractors of experience and capital cannot be found, a system of day-work must be adopted, with a staff of pay-clerks, inspectors, &c., and the plant also; even the simplest tools must be provided from the public grants; so that a very small proportion of the total cost is applied, in wages, to that class for whose support the whole was at first undertaken.

The calculation of the dimensions of what are called equivalent, or equal-discharging channels, has evidently, from this and the last example, some practical value: and, first, of those which have a rectangular transverse section. Suppose, then, it is required to determine the width of a channel, having a given depth, which shall discharge the same volume of water as another, whose dimensions and discharge are known, the rate of fall being the same in both.

Let p be the given depth, x the required width. Then $0.92\sqrt{2f\frac{px}{2p+x}}\times px = Q$, the discharge, or, omitting $0.92\sqrt{2f}$ as common to all, we have

$$\sqrt{\frac{px}{2p+x}}\times px = C,$$

and

$$\frac{p^3 x^3}{2p+x} = C^2, \text{ hence } x^3 - \frac{C^2}{p^3}x = \frac{2C^2}{p^2},$$

which may readily be solved by a few substitutions, and the aid of Tables of squares and cubes. In order to construct the diagram, Fig. 69 *a*, comparable to that in Fig. 69, and calculate a Table, the rectangle $A'D'D_1B_1$, of depth 50, and breadth 100 units, at the same scale as the former, was taken as a standard of comparison, and the other several equal-discharging channels plotted to the same scale, and having the vertical centre line CD in common, the resulting curve $d'D'd''\,d_1D_1$ is analagous to that of the rectangle of constant area, which is repeated in dotted lines in Fig. 69 *a* for the sake of comparison.

If the breadth b were given to find the depth z, we have

$$z^3 - \frac{2C^2}{b^3}z = \frac{C^2}{b^2}.$$

TABLE *giving the relative dimensions of equal-discharging channels, having rectangular transverse sections*, Fig. 69*a*.

Depth.	Breadth.	Border.	Area.	Hydraulic mean depth	Square root of hydraulic mean depth.
10	801.10	821.10	8011.0	9.75	3.12
20	297.69	337.69	5953.8	17.63	4.20
30	176.15	236.15	5284.5	22.40	4.73
40	126.30	206.30	5052.0	24.50	4.95
50	100.00	200.0	5000.0	25.00	5.00
100	54.20	254.20	5420.0	21.32	4.61
150	39.79	339.79	5968.5	17.60	4.19

In the two cases, shown in Fig. 69, we have the area of the rectangle whose depth is equal to twice the width, either a maximum or constant; in this we perceive it is a minimum.

In channels of a trapezoidal transverse section, if the mean breadth be great compared with the depth, we find that the rectangle of equal area, and the same depth (in which case the mean breadth of the trapezium is equal to the width of the rectangle), is very nearly of equal discharge, the fall being the same in both, and hence we can approximately determine equivalent trapeziums.

Let there be a trapezium of 5 ft. depth, and a mean breadth of 25 ft., to determine the dimensions of another of equal discharge, and 7 ft. deep, the mean breadth is therefore $125 \div 7 = 17.86$ ft.; and if through the middle point of the side of this last rectangle we draw lines inclined at the inclination of the side slopes of the given trapezium, we shall obtain approximately that of equal discharge.

Let us calculate for each, and suppose that n, the ratio of the slopes, is 2 to 1, then the trapezium of 5 ft. depth and 25 ft. mean width has a bottom width of 15 ft., and the side slopes equal to $p\sqrt{n^2 + 1}$, (p. 136), are $5 \times 2.236 = 11.18$ ft. each, and the border consequently is $15 + 22.36 = 37.36$ ft., and the hydraulic mean depth 3.343; as before, omitting from each $0.92 \sqrt{2f}$ as common to both, then $\sqrt{3.343} \times 125 = 228.75$ will represent a number proportional to the discharge, or the quantity to be compared in each trapezium, and which should be nearly equal for both.

The trapezium of 7 ft. depth, and $n = 2$, has a bottom width of $17.86 - 14 = 3.86$ ft., and side slopes of $2 \times 7 \times 2.236$

= 31.3 ft., so that the border will be 35.16 ft., and hydraulic mean depth 3.555 ft., and $\sqrt{3.555} \times 125 = 235$, is the number which we must compare with 228.75, the result being, for the channel of 7 ft. deep, a discharge 2.7 per cent. too great.

The exact determination of the bottom width *x*, of equal-discharging trapeziums, of depth *p*, we derive from the expression

$$\sqrt{\frac{px + p^2n}{2p\sqrt{n^2 + 1} + x}} \times \overline{px \times p^2n} = C,$$

hence

$$x^3 + 3pnx^2 + \left(3p^2n^2 - \frac{C^2}{p^3}\right)x = 2C^2\,\frac{\sqrt{n^2 + 1}}{p^2} - p^3n^3,$$

or with the numerical data above,

$$x^3 + 42x^2 + 435.445x = 2028.4,$$

which is satisfied by $x = 3.42$ very nearly. For constructing a trapezium with that base, and having a depth of 7 ft., and side slopes two to one, we find the area 121.94, and the square root of hydraulic mean depth 1.874, giving $228.5 = C$.

However well designed and executed in every respect the new and rectified channels of a river may be, the lapse of time would, in every case, destroy the form, and alter the directions of the bed, the same causes being still in action that had given the original course its defective character. Trustees were, therefore, appointed for each river by Act of Parliament, whose duty it is to visit the completed works every year; and power is given to them to order all needful repairs, charging the improved lands with the requisite outlay.

(LXI.). Two very distinct systems of house-drainage are now to be found in use in towns and cities—one, that of the cess-pool, with a periodical clearing out of its contents; in the other, the discharge of all refuse, or, as it is now called, sewage, from the houses, takes place by drain-pipes into street-sewers, which, either singly, or after the union of several into one large main sewer, are discharged into the river, or natural out-fall of the district.

It is to obviate the yearly increasing pollution of the Thames, arising from this last cause, that the great lines of sewers of the Metropolitan Board of Works are now being carried out. In London, within the last few years, cess-pools have, from a conviction of their unhealthiness, been interdicted in all new structures, and closed up where heretofore in use;

the second of the systems above-mentioned being now compulsory.

However beneficial this change may have been in the single point of obviating the unhealthy effluvia from the cesspool and the saturated ground around it, other evils of great magnitude were unexpectedly found to arise. For the existing lines of main sewers all discharge into the Thames, and, having courses nearly perpendicular to it, deliver off their contents within the bounds of the city; the ebb and flow of the tide tends, moreover, to keep the foul water many days, oscillating as it were, before the points of discharge, the mouths of these sewers, and all the longer in the dry summer season, when the volume of fresh water coming down the river is diminished, and the high temperature of the air favours putrescence. At low-water, also, the river banks, covered with an offensive deposit of the sewage, are exposed for many hours every day, greatly increasing the evil. These injurious consequences appear to increase, not only from the growth of the city; but the great water companies, chiefly supplied from the Thames, a few miles up stream, draw each year a larger quantity from the river,—a quantity which, in dry seasons, is no inconsiderable portion of its total discharge. Now, a large part of the supply of water to houses is carried eventually into the street-sewers, and so into the river, in the state of sewage; thus the pure water coming down the river, at the critical time when it is most required to dilute and carry down seawards the sewage of the city, is materially diminished, and the sewage itself proportionally increased.

The river Thames was, in fact, by this system, made a great open sewer through the centre of the city, which was year by year increasing in the offensiveness, not only of the water, but also of the banks exposed each ebb tide; and even this injurious mode of drainage was becoming inadequate. In Mr. R. Stephenson's words:—"All these sewers are becoming daily more charged with sewage, in consequence of the increase of building upon the areas they drain; and their inadequacy to give vent to the flood-waters is still more apparent; for, as the open lands and gardens are becoming occupied by dwellings, the area upon which flood-waters are retained, and allowed to flow off gradually, is sensibly diminished. The natural consequence of this change of circumstances is, that the floods descend with increased rapidity, and gorge the main sewers in the lower districts towards the Thames, in some instances far beyond their capacity of discharge."

"These alterations of circumstances, arising out of the extension of the area upon which buildings are daily growing up, cannot be looked upon as having reached their limit. We must assume, therefore, that large portions of the above districts will go on increasing in population; and the sewers in the lower levels, originally abundantly large, must shortly become wholly insufficient."—*Report*, 11th December, 1854.

Paris, again, is a city in which the cess-pool system is most fully carried out, with the unavoidable concomitant of carting away the contents, which are deposited outside the suburbs, and do not ever reach the river Seine. The street-sewers only carry to the river the rain-fall, the waste of fountains, &c.

After many years, and the discussion of several different plans, it has been determined to construct, on the north side of the Thames, through London, three lines of main sewers, the high, middle, and low levels, not running in directions perpendicular to the river, but on the contrary nearly parallel to it; these, uniting into one, near the crossing of the river Lea, are carried many miles down, and eventually discharge into the river Roding, at Barking Creek, a point so distant from London, that it is fully expected the flood-tide cannot convey up the Thames, towards the city, any of the sewage discharged into it by the branch river above mentioned. On the south side, the same system has been adopted; several lines, parallel to the general direction of the river, unite into one, and are conveyed down stream to a distant out-fall.

In these great works of main and intercepting drainage, they have adopted, in the upper parts of the main lines, a transverse section for the brick-work sewers, shown in Fig. 101, and evidently very similar to that of the wrought-iron tube, Fig. 95, mentioned, pp. 181, 182.

It is thus designed: two circles, Fig. 100, having their centres A and B in the same vertical line, are drawn to touch each other, the upper having a radius twice as great as the lower; a horizontal line being drawn through A, an arc of a circle, whose centre is in this line, is drawn, touching both the circles, and this construction being repeated on each side, the contour of the inner lines of the sewer is obtained. The thickness of the brick-work in every part being laid off on the continuation of the several radii, the external figure is determined.

This oval, or egg-shaped form, was adopted after much discussion. It had been found by experience that those sewers which had a flat invert, and considerable width below, with nearly vertical sides, deposited the sewage matter, through which, in the ordinary states of the flow, a small and inoperative current wound its way. The improved design, being, on the contrary, narrowed down in the lower part, concentrates the scouring action, even when the flow may be at a low point; should the quantity to be discharged greatly increase from storms of rain, as it does very suddenly in towns, the transverse section is also increased as the level of the water rises, and possesses a large discharging power. As, however, the depth is, in this form of sewer, 1½ times the breadth, it would be inconvenient and unsuitable beyond a certain breadth. In the middle-level sewer, which commences at the Great Western Railway, and is carried through London, for 9½ miles, to its junction with the high and low level sewers, near the river Lea, the oval or egg-shaped sewer commences with a depth of 4 ft. 6 in. and breadth 3 ft.; and gradually increasing by 3 in. at a time, reaches a depth of 6 ft., and breadth of 4 ft., after which a circular transverse section is adopted; this also being gradually enlarged up to 10 ft. diameter.

It is necessary, in giving designs in detail for a work of city drainage, to form, first, an estimate of the quantity of fluid to be conveyed from a given district, having a well-defined boundary and density of population, and afterwards assign the least velocity, so that no depositions of solid sewage matter take place, and from these decide on the detail of the declivity and transverse area.

The amount of sewage proposed to be intercepted from the more closely inhabited districts, has been estimated upon data which make provision for the increase of future generations. Thus, where the population, as at present existing, exceeds 30,000 to the square mile—and it is at the rate of 80,000, and even 100,000, in some localities—the actual numbers are estimated for; and where it is less, allowance has been made for that number, in order to provide for future increase. The supply of water per head has been reckoned at 5 cb. ft. per diem (31¼ gallons) for each individual, being on a more liberal scale than now actually obtained; and of this quantity, it is contemplated that one-half will pass away during 6 hours of the day (generally from 6 A. M. to noon), $2.5 \times 30{,}000 \div 640 \times 360 = 0.327$ cb. ft. per acre per minute.

It had been determined, by lengthened and careful observation, that the least velocity of the current which suffices to prevent deposit in sewers is 150 ft. per minute, about 1¾ miles per hour; such being the case, we perceive one element in the design of such works, namely, that the rate of fall and dimensions of the transverse sections of the sewer must be so related to each other, as that the velocity be not less than this, and as the sewer must also be smaller near its commencement;

in order to be proportioned to the quantity it has to convey, it follows that the inclination should be greater at the upper end of the line, diminishing as it proceeds downwards, and so graduated as to produce a uniform rate of flow, and thus avoid a waste of fall in one portion of the design, and an insufficient velocity in another.

But in deciding on the rate of fall in the different parts, the engineer is restricted within narrower limits than in perhaps any other works; so that it will be necessary, first, to determine, from the longitudinal section, the practicable limits which can be assigned to the declivity in the different parts of the course; and then, from the velocity, determined on as above, it becomes easy to decide on the size of sewer requisite to carry off a constant and well-defined quantity of sewage in a given time.

Mr. Robert Stephenson and Mr. Hawkesley have reported it as their opinion, that a velocity of 2 miles per hour, or 176 ft. per minute, was ample for sewers in which there is a continual stream. As to the determination of the declivity necessary to cause this velocity, Mr. Bazalgette, engineer to the Metropolitan Drainage Commission, writes in his Report (3rd April, 1856): "With reference to the fall requisite to produce this velocity, calculations, based on the principles universally adopted, show that in a sewer 9 ft. 6 in. in diameter, with a fall of 2 ft. per mile, the mean velocity would be about 2 miles per hour, when running three-quarters-full, or 1.88 miles per hour when running half-full, and about 1⅔ miles per hour when running one-third-full.

"But the velocity of a stream is governed by the dimensions of its channel, as well as by its rate of fall. Thus, in a sewer 10 ft. in diameter, with a fall of 2 ft. per mile, there will be the same mean velocity of stream, when running half-full, as in a sewer 2 ft. in diameter with a fall of 10 ft. per mile, also half-full."

But, besides this quantity of sewage, a certain amount of rain-fall must also be intercepted; and various reasons led to the estimate of a quarter of an inch of rain in 24 hours, over the area of urban districts: as a quarter of an inch is equal to 0.0208 ft., and an acre to 43,560 sq. ft., this amounts to 906 cb. ft. per acre in 24 hours, or 0.63 cb. ft. per acre per minute, which added to the former quantity, we have $0.327 + 0.63 = 0.957$, very nearly 1 cb. ft. per minute per acre.

Extraordinary storms, and rain-falls exceeding the above, are not carried down the total length of the new main-sewers, but are delivered off into the existing lines of sewers by lateral openings of considerable length, in the circular sewers, the sill of which is placed above the half-depth. Thus the present main-sewers, which have acted to carry to the Thames in nearly direct lines, perpendicular to its course, both the drainage and sewage, will, on the completion of the main intercepting sewers, chiefly discharge but the overflow of storms of rain, whatever sewage matter it may contain being necessarily in a very diluted state.

The real difficulty and expense of such works as these arise from the uncertain amount of rain-fall to be provided for in great storms. The house-drainage, though it may in the greater part flow down in a few hours, conforms very steadily to a known average; but the former, though a quarter of an inch in 24 hours may be the value of its average, has yet reached 3 in. in 4 hours in London, and this must have been discharged nearly in that time. If we take 1 in. per hour, the discharge amounts to 60 cb. ft. per acre per minute.

The ingenious device of lateral openings, placed so high as to come into operation only when the storm-flow has reached a considerable depth, gives safety to the new sewers, and makes use of the present main lines in a manner well suited to their great section and direct course to the river.

It becomes a matter of great interest, therefore, to calculate the different rates of discharge and velocity in circular sewers for each different depth of flow, supposing always that it has attained a uniform rate of motion. In the following Table the vertical diameter is supposed divided into 16 equal parts, radius being unity. The curious fact is at once evident from an inspection of the columns headed Hydraulic Mean Depth, and Square Root of Hydraulic Mean Depth, that the maximum velocity, which is a function of that quantity, is at a depth of $\frac{13}{8}$ths of the radius, or $\frac{5}{8}$ths above the centre. And that the discharge, which is a function of the product of the area into the square root of the hydraulic mean depth, is not a maximum when flowing full, but at a depth of $\frac{15}{8}$ths, or $\frac{7}{8}$ths above the centre, as is shown by an inspection of the figures in the last column. In fact, in this last case the additions to the area are relatively less than those of the corresponding portions of the circumference, that is of the border, in the higher parallels through the several eighths into which the radius is supposed to be divided. The investigation was made by the Rev. J. A. Galbraith, Professor of Natural Philosophy in this University.

TABLE *for calculating the Velocity and Discharge of Water flowing in Pipes, not under pressure, at the depths specified. Radius* = 1.

Depth. Rad. = 1	Border.	Area.	Hydraulic mean depth.	Square root of hydraulic mean depth.	Area into sq. root of hydraulic mean depth.
$\frac{1}{8}$	1.0105	0.0817	0.0809	0.2844	0.0232
$\frac{2}{8}$	1.4451	0.2266	0.1568	0.3959	0.0897
$\frac{3}{8}$	1.7913	0.4077	0.2276	0.4771	0.1945
$\frac{4}{8}$	2.0944	0.6142	0.2932	0.5415	0.3326
$\frac{5}{8}$	2.3728	0.8368	0.3527	0.5938	0.4969
$\frac{6}{8}$	2.6360	1.0759	0.4081	0.6389	0.6873
$\frac{7}{8}$	2.8908	1.3214	0.4571	0.6761	0.8934
$\frac{8}{8}$	3.1416	1.5708	0.5000	0.7071	1.1107
$\frac{9}{8}$	3.3923	1.8202	0.5366	0.7325	1.3333
$\frac{10}{8}$	3.6471	2.0656	0.5664	0.7526	1.5545
$\frac{11}{8}$	3.9104	2.3048	0.5894	0.7677	1.7695
$\frac{12}{8}$	4.1888	2.5274	0.6034	0.7768	1.9632
$\frac{13}{8}$	4.4919	2.7338	0.6086	*0.7801	2.1327
$\frac{14}{8}$	4.8380	2.9150	0.6025	0.7762	2.2626
$\frac{15}{8}$	5.2726	3.0598	0.5803	0.7618	*2.3310
$\frac{16}{8}$	6.2832	3.1416	0.5000	0.7071	2.2214

To apply the Table—which is adapted to any dimensions as feet, inches, mètres, &c.—let us suppose the diameter of a circular culvert to be 8 ft.; the radius, therefore, which is unity in the Table, is in this case 4, and all lineal dimensions, as in the columns border and hydraulic mean depth, must be multiplied by that number to obtain their value at 8 ft. diameter; the column area must be multiplied in like manner by 4^2, that of square root of hydraulic mean depth by $\sqrt{4}$, and the last column by $4^2\sqrt{4}$, and so of any other radius—the two last columns thus giving numbers which are proportional to the velocity and the discharge, respectively; and when these are multiplied again by $0{\cdot}92\sqrt{2f}$, give the actual velocity and discharge at the several depths assumed. If, then, this culvert, 8 ft. in diameter, be running half-full, we find from the horizontal column 8-8ths, which answers to that depth, border $3.1416 \times 4 = 12.5664$ ft., area $1.5708 \times 4^2 = 25.1328$ sq. ft. Hydraulic mean depth $0.5 \times 4 = 2$ ft. The square root of the same, $0.7071 \times \sqrt{4} = 1.4142$. The last column, $1.1107 \times 4^2 \times \sqrt{4} = 35.54$. If, again, we suppose the fall to be 2 ft. per mile, we have both the velocity and the discharge, by multiplying these two last numbers into $0.92\sqrt{2 \times 2} = 1.84$. The sewer, 8 ft. in diameter, running half-full, and with a declivity of 2 ft. per mile, has therefore a mean velocity 1.4142×1.84

$= 2.6$ ft. per second, or 156 ft. per minute. The discharge will be $35.54 \times 1.84 = 65.4$ cb. ft. per second, or 3924 cb. ft. per minute, which result is also given by 25.1328×156.

Again, suppose the depth in the culvert was $\frac{3}{4}$ths of the diameter, answering to the line $\frac{12}{8}$ths. From it we have the border equal to $4.1888 \times 4 = 16.7552$ ft.; the area $2.5274 \times 16 = 40.44$ sq. ft.; the hydraulic mean depth and its square root $0.6034 \times 4 = 2.4136$ ft., and $0.7768 \times \sqrt{4} = 1.5536$ ft. respectively. The discharge is given at once from the tabular number under area into square root of hydraulic mean depth, that is $1.9632 \times 4^2 \times \sqrt{4} \times 1.84 \times 60 = 6935.6$. The number 1.84 being $0.92 \times \sqrt{2f}$ as before.

Again, if the diameter be supposed 6 ft., we must multiply by $3^2 \times \sqrt{3} = 15.65$, the several numbers in the last column on the right hand side, according to the fraction of the depth at which the current is supposed to be flowing; and thus if the depth be taken at $\frac{12}{8}$ths, and the fall per mile 2 ft. as before, we have $1.9632 \times 15.65 \times 1.84 \times 60 = 3392$ cb. ft. per minute.

It was said above that a sewer of 9 ft. 6 in. in diameter, with a fall of 2 ft. per mile, and running $\frac{3}{4}$-full, would be about 2 miles per hour; when half-full, 1.88 miles per hour. In the column headed sq. root of hydraulic mean depth, and on line $\frac{12}{8}$th, we have the tabular number 0.7768; this into $\sqrt{4.25} = 2.06$ and $0.92\sqrt{2f} = 1.84$, giving 2.944 ft. per sec., or 2.002 miles per hour.

In order to turn feet per second into miles per hour, we have only to multiply by 0.68, that is $3600'' \div 5280$ ft.

When half-full, 2.68 ft. per second, or 1.82 miles per hour. We can by this Table also examine the other quotations given above, namely, that the mean velocity of a sewer, running half-full and 10 ft. diameter, and fall 2 ft. per mile, is the same as that of a sewer 2 ft. diameter, and 10 ft. fall, also running half-full. In the first we have tabular number $0.7071\sqrt{10} \times (0.92\sqrt{2 \times 2})$, in the second $0.7071 \times \sqrt{2} \times (0.92\sqrt{2 \times 10})$, and these are evidently equal. The discharges in these two cases would be as the squares of the diameters, since the velocities are equal. The result would be equally true for any other fractions of the depth.

In the Table, if the subdivisions of the diameter had been carried further, it would have been found that the maximum discharge was at nearly $\frac{19}{20}$ths of the diameter, not $\frac{15}{8}$ths; but the maximum velocity is very nearly $\frac{5}{8}$ths of the radius above the centre.

Fig. 72 represents the section of a circular culvert, or sewer, the circumference being supposed to represent its inner boundary. The vertical diameter is divided into 16 equal parts; and horizontal lines drawn through these points, representing the water-surface at the several eighths of the radius. The dotted line, *ab*, is the level of the maximum discharge, and A, B, which coincides very nearly with the horizontal at $\frac{13}{8}$ths, is the level which has the greatest velocity; the angle at the centre, subtended by the chord A, B, being 102°, 32′, 48″. Fig. 72 *a* is a diagram representing, as in Figs. 70 and 71, a curve which shows the value of the hydraulic mean depth at any depth of current, C, D, being equal to the diameter C, D, Fig. 72, and divided into the same number of parts. It will be perceived that the ordinates at 8 and 16 are equal; that is, the current, running full, and half-full, has the same hydraulic mean depth, which is equal to half the radius. The maximum value is drawn with a strong line intermediate, but not equidistant, between the equal values. Any depth being taken between 8 and that which gives the maximum, another between the maximum and 16 may be assigned which will have the same hydraulic mean depth, and therefore same velocity, *cæteris paribus*.

(LXII.) If any obstruction has been placed in the channel of a river, such as a weir, or the piers of a bridge, &c., the water is impeded in its flow, and compelled to pass through a reduced area of transverse section; the necessary consequence of any arrangement of this nature is to give the water a higher velocity at this part. But this result, again, could not follow, unless the surface of the water had risen at the reduced area, in order to create a "head," or "charge," sufficient to produce this excess of velocity which the current must take. We find two very distinct questions arise, connected with the resulting change in the water surface—

First, the vertical rise at the immediate locality of the impediment; and, secondly, the elevation of the surface of the water consequent on this at definite distances up stream, which last indirectly determines the furthest length to which the backwater extends.

We may distinguish three classes of barrier rendered necessary by works in connexion with rivers; first, the stream is completely dammed up, Fig. 111, and the discharge takes place by a sluice of given area and position, relative to the bottom line of the river. Secondly, the water is discharged, Figs. 107, 108, 109, freely over the upper edge, or crest, of a weir. And, thirdly, we have obstructions which block up only a part of the water-way, such as the piers of a bridge, Fig. 110; or, as is common on the rivers of the Continent, a mass of masonry is carried from one bank of a river across a part only of the bed, leaving a narrow channel at the opposite side, in which, though the velocity is increased, the depth for purposes of navigation is improved.

The immediate rise in the first case we obtain from the formula, § 18, p. 15, squaring and transposing,

$$H = \frac{Q^2}{S^2} \times 0.04065.$$

This total elevation may be divided into two distinct parts, one depending on the construction, namely, the height of the sill of the sluice opening above the bottom line of the bed, the other on the volume Q, which the stream delivers; and is independent of the rate of surface-fall, or velocity of the river course above the point.

In the second case we have from § 55, eliminating H,

$$H = \sqrt[3]{\left(\frac{Q}{3.53l}\right)^2} = 0.429\sqrt[3]{\left(\frac{Q}{l}\right)^2};$$

and this, in like manner, depends on the height of the crest and volume of water, being independent of the surface-fall.

In the third case, of the bridge, or partial dam, let L be the mean width of the river above the contracted part, and V the mean velocity in it, S being the transverse area, and l, v, and s the corresponding values at the contracted section; the volume passing down in each case being Q. The depth at s being p, and that at S being $p + x$, representing by x the fall produced by the contraction of the channel, that is AX, Fig. 110.

Now the vertical heights, or heads, necessary to produce the velocities v and V, are given by the expressions $\frac{v^2}{2g}$ and $\frac{V^2}{2g}$, so that x being the difference between these heads, we have

$$x = \frac{v^2}{2g} - \frac{V^2}{2g},$$

moreover as

$$v = \frac{Q}{s} \text{ and } V = \frac{Q}{S},$$

we have

$$x = \frac{Q^2}{2g}\left(\frac{1}{s^2} - \frac{1}{S^2}\right),$$

and substituting for s and S their values considered as rectangles of the width l, this becomes

$$x = \frac{Q^2}{2g}\left(\frac{1}{m^2l^2p^2} - \frac{1}{L^2(p+x)^2}\right),$$

in which m has the same signification as in the sections 14 and 54, on orifices and weirs, and is estimated at 0.90 to 0.80, according to the form of the cut-water. Eliminating x, we have a cubic equation, or we may substitute values for it until both sides are equal.

Thus we obtain, in all three cases, the vertical rise at the immediate locality of the obstruction in the river channel.

Our next object must be to assign the vertical rise of the surface of the water consequent on it, at given distances up stream, and thus indirectly determine the furthest length, or amplitude, of the back-water; and it is now that the surface-fall and velocity influence the result. One of the earliest surmises as to the curve of the surface was, that it coincided with an arc of a circle, touching the horizontal line AN, Fig. 105, drawn through A, the point of the surface, where it begins to curve down to, or over the discharging orifice or weir, and also the line of the original surface of the stream MR atM.

The distance AN has been technically called the hydrostatic, and ANM the hydraulic amplitude, the latter being evidently double the former. If the back-water were simply a wedge superimposed on the original surface of the stream, and uninfluenced by its motion, it would extend horizontally from A to N, and its value would be $x \div \sin i$, the angle of inclination of the original surface of the stream being i, and x the rise at A, consequently $AM \doteq 2\left(\frac{x}{\sin i}\right)$ nearly. Instead of using the coefficient 2, some authors propose 1.9, or 1.5, the latter being probably a very safe mean.

Having thus, in all these different cases, obtained the depth at the transverse section AXY, which we will call p_0, we can proceed to calculate the length up stream z_0, at which the depth will be equal to any assigned quantity p_1, taken necessarily between the extreme p_0, and the depth at the point M, where the stream, as yet uninfluenced by the restriction of its bed, has a uniform velocity and constant depth, which we may call P; and from the surface-line, thus obtained, determine the depth at a given distance.

The channel may be taken as of uniform width, equal to w, and rectangular transverse section; so that the only alteration that takes place in the several transverse sections up stream is in the value of the depths p_0, p_1, &c.

Let A′X′Y′ be the position of the transverse section, having the assigned depth p_1, at the unknown distance z_0 from AXY, and so on successively for the distances z_1, z_2, between the other assigned depths p_2, p_3, &c.

We may suppose the curved surface of the stream to be a right line in the several distances between the points A and A′, &c.

Now, as the velocities in the transverse sections are inversely proportional to their areas (§ 84), if we put v_0 and v_1, for the velocities at the sections which have the depths p_0 and p_1, we have

$$v_0 : v_1 :: p_1 : p_0,$$

and may therefore express v_1, in terms of v_0, and the respective depths, that is $v_1 = v_0 \times \frac{p_0}{p_1}$, and as the heads, or vertical heights necessary to produce the velocities v_0 and v_1 are $\frac{v_0^2}{2g}$ and $\frac{v_1^2}{2g}$, which last is equal to $\frac{v_0^2 \frac{p_0^2}{p_1^2}}{2g}$, we have

$$\frac{v_1^2}{2g} - \frac{v_0^2}{2g} = \frac{v_0^2 \frac{p_0^2}{p_1^2} - v_0^2}{2g} = \frac{p_0^2 - p_1^2}{p_1^2} \times \frac{v_0^2}{2g} \qquad (1)$$

But again, the resistance from the friction of the bed in the several length z_0, z_1, &c., requires a certain head to overcome it. In order to give the expression for this resistance a manageable degree of simplicity, we may, without error, assume that the decreasing velocity in the length z_0 is constant, and instead of taking it as the mean $\left(\frac{v_0 + v_1}{2}\right)$ between v_0 and v_1, put it as equal to that in the transverse section AXY. In like manner for the hydraulic mean depth, instead of the mean between those of the sections at AXY, and at A′X′Y′, we may represent it by that of the former, namely, $\frac{p_0 w}{2p_0 + w}$, which, using a former notation (p. 133), is expressed by H_y.

If in the equation (c), page 126, namely, $\frac{h}{l} = \frac{a}{g} \times \frac{c}{s} \times v^2$, we put for $\frac{c}{s}$ its value $\frac{1}{H_y}$, and multiply both sides by l, we have

$$h = 2a \times \frac{l}{H_y} \times \frac{v^2}{2g}, \tag{2}$$

having also multiplied the right-hand side, above and below, by 2. In this expression the velocity v is supposed to have become uniform, and therefore necessarily the resistances exactly equal to the force producing motion. If we substitute z_0 for l, and v_0 for v, the velocity in z_0 being supposed uniform, this becomes

$$h = z_0 \times 2a \times \frac{1}{H_y} \times \frac{v_0^2}{2g},$$

which gives the value of the head, or pressure, necessary to overcome the friction arising from the uniform velocity v_0, in a channel of the length z_0, and hydraulic mean depth H_y. And therefore the total resulting alteration of head H, between the points A′ and A, will be represented by

$$H = z_0 \times 2a \times \frac{1}{H_y} \times \frac{v_0^2}{2g} - \frac{p_0^2 - p_1^2}{p_1^2} \times \frac{v_0^2}{2g}, \tag{3}$$

that is the difference between (1) and (2), which have opposite effects on the inclination of the surface of the water; the former, or positive quantity, tending to heap up or raise the surface; the latter, that is the gradual increase of transverse area, to diminish it.

From the point A′, Fig. 75, draw a horizontal line perpendicular to AXY, and cutting it produced in the point N, and also from the same point a parallel A′R to the line of the surface X′X of the stream before it had been impeded, and which is by supposition parallel to the line of the bottom bed Y′Y.

The line NA will then represent the resulting alteration of head between the transverse sections at A′ and A, the value of which was given in (3), and AR is obviously equal to $p_0 - p_1$.

Again, i being the angle of inclination of X′X to the horizon, NR is equal to $z \sin i$, so that

$$\text{NR} - \text{AR} = \text{NA}$$

$$= z_0 \sin i - \overline{p_0 - p_1} = z_0 \times 2a \times \frac{1}{H_y} \times \frac{v_0^2}{2g} - \frac{p_0^2 - p_1^2}{p_1^2} \times \frac{v_0^2}{2g}. \tag{4}$$

Transposing and changing signs, we have

$$p_0 - p_1 = z_0 \sin i - z_0 \times 2a \times \frac{1}{H_y} \times \frac{v_0^2}{2g} + \frac{p_0^2 - p_1^2}{p_1^2} \times \frac{v_0^2}{2g}. \quad (5)$$

In order to have all the terms of the right-hand side multiplied by z_0, transpose and arrange so that, as $p_0 - p_1^2 = \overline{p_0 - p_1} \times p_0 + p_1$, we have

$$\overline{p_0 - p_1} \times \left(1 - \frac{p_0 + p_1}{p_1^2} \times \frac{v_0^2}{2g}\right) = z_0 \times \left[\sin i - 2a \times \frac{1}{H_y} \times \frac{v_0^2}{2g}\right]. \quad (6)$$

Now as $\frac{p_0 + p_1}{p_1^2}$ is very nearly equal to $\frac{2p_0}{p_0^2}$, or $\frac{2}{p_0}$, equation (6) becomes by dividing

$$p_0 - p_1 = z_0 \times \left\{\frac{\sin i - 2a \times \frac{1}{H_y} \times \frac{v_0^2}{2g}}{1 - \frac{2}{p_0} \times \frac{v_0^2}{2g}}\right\}. \quad (7)$$

And again, multiplying the fractional coefficient of z_0, above and below, by p_0,

$$p_0 - p_1 = z_0 \left\{\frac{\sin i \, . \, p_0 - 2a \times \frac{p_0}{H_y} \times \frac{v_0^2}{2g}}{p_0 - 2 \times \frac{v_0^2}{2g}}\right\}. \quad (8)$$

Eliminating z_0, and multiplying above and below by $2g$, we have

$$z_0 = p_0 - p_1 \times \left[\frac{p_0 \times 2g - 2v_0^2}{\sin i \, . \, p_0 \, . \, 2g - 2a \, . \, \frac{p_0}{H_y} \, . \, v_0^2}\right]. \quad (9)$$

From which we may calculate the length up stream at which the assigned depth p_1 will be found.

Examining this value of z_0, we perceive that the fractional coefficient of $\overline{p_0 - p_1}$ on the right-hand side may become equal to zero, to infinity, or to unity, or negative; and therefore z_0 also become zero, infinite, or equal to $p_0 - p_1$, or negative, and this without any of the separate variable factors in the fraction becoming equal to zero, or infinite; inasmuch as both the numerator and denominator consist of two parts connected together by the negative sign.

And first of the numerator, $p_0 2g - 2v_0^2$, it is evident that if $2v_0^2$ be equal to $p_0 2g$, its value will be zero, and consequently (if the denominator be not simultaneously equal to zero) that of z_0 also; or, in other words, the circumstances of the stream as to velocity and inclination do not admit of any true back-water being formed. Now if $p_0 2g = 2v_0^2$, we have

$$\frac{p_0}{2} = \frac{v_0^2}{2g}. \qquad (10)$$

Indicating that when the velocity of the water is such that the head necessary to produce it is equal to half the depth of the stream, the distance of the assigned depth p_1 from p_0 will disappear, however different p_1 may be from p_0, or become negative if $\frac{v_0^2}{2g}$ be greater than $\frac{p_0}{2}$.

Fig. 106, quoted from D'Aubuisson, represents a longitudinal section of an experimental channel, the stream flowing in which had this relation of depth to velocity. The resulting surface of the water is characterised by wholly different, and nearly opposite, circumstances to that of the ordinary back-water,—being convex to the line drawn through A, parallel to the original surface of the stream, and towards the end very quickly so, being separated from the general surface of the stream by an abrupt and recurved termination. The length, also, is far less than that of the hydrostatic amplitude.

Such curvatures of the surface can evidently only occur in water-courses of great velocity and of very small depth, and such are rarely found in nature for any considerable length.

Again, it is evident that if the two parts of which the denominator $\sin i . p_0 . 2g - 2a . \frac{p_0}{H_y} . v_0^2$ consists become equal, the fraction itself, and consequently z_0, becomes infinite (provided the numerator be not simultaneously equal to zero). If, then, in the equation

$$\sin i . p_0 . 2g = 2a \frac{p_0}{H_y} . v_0^2, \qquad (11)$$

we eliminate v_0^2, we have, taking the root,

$$v_0 = \sqrt{\frac{g}{a}} \times \sqrt{\sin i . H_y}; \qquad (12)$$

and as (p. 126) $\frac{a}{g} = a'$, and the value of a', deduced from experiment, is found (p. 127) to be 0.0001 nearly, we have $a = 0.0001 \times g = 0.0032$, so that equation (12) is evidently identical with (e) p. 127, when for sin i we put its equal $\frac{h}{l}$, and for H_y its value $\frac{S}{C}$, being tantamount to this in words, that when the mean velocity in the length of the channel z_0, approaching the transverse section AXY, is the same as that which is found to obtain in a uniform stream of the same surface inclination and transverse dimensions, then the length of the back-water will extend up indefinitely, or so far as the stream in which the obstructing works are placed continues unaltered in slope and dimensions.

If, again, the numerator and denominator become zero simultaneously, then the fraction becomes $\frac{0}{0} = 1$, and $z_0 = p_0 - p_1$, and therefore from the numerator we have, eliminating v_0^2 in equation (10), $v_0^2 = p_0 g$, and, in like manner, from the denominator $v_0^2 = \frac{\sin i \,.\, H_y \,.\, 2g}{2a}$, so that, equating these two values, we have $\sin i = a \times \frac{p_0}{H_y}$, showing that, in this case, the inclination of the surface of the river, measured by its sine, is equal to the constant 0.0032 multiplied into the ratio of the depth to the hydraulic mean depth of the part contiguous to the section AXY.

In rivers which are wide in proportion to their depth, we know (p. 132) that the hydraulic mean depth is very nearly equal to the depth, and then $\sin i = 0.0032$, or $\frac{1}{313}$ nearly; and, conversely, we perceive that if any channel have this slope of about 1 in 313, and having also a uniform transverse section, and therefore constant velocity and surface parallel to the bed: then, if at any point the stream fall vertically over an edge, of the same length as the breadth of the channel, we shall find that no alteration of the surface-inclination takes place up stream; it will continue, as before, parallel to the line of the bed.

The above formulæ for the back-water have been compiled from what has been given upon this subject by D'Aubuisson, and also by Mr. Neville, in his Hydraulic Tables.

The suppositions which are introduced into the computation of z_0, &c., to simplify the expressions, are productive of no error of an appreciable character. It is the assumption of a uniform channel above the site of the impeding works, uniform, not only in breadth, but in the rate of fall also, that really removes the calculation from perfect identity with the cases of back-water which occur in rivers; for they are ever found subject to abrupt changes in width and surface-inclination.

It would not be difficult, however, by an accurate map and section of a river, to divide the bed into portions, in which the width and inclination may be taken as uniform; and then, assuming depths p_1, &c., until tentatively the resulting length z_0, &c. coincide with the observed changes in the river-course, to calculate for each different part separately, and to decide as to the amount of injurious back-water which may be caused at any point up stream.

In order to trace more clearly the mutual relations of the several parts of the formula, let us suppose a river of given dimensions, discharge, &c., and observe the increase or decrease of the numerical values of the terms, as we calculate the successive lengths z_0, z_1, &c., from the assigned depths.

Thus, suppose a river of a uniform mean breadth w equal to 70 ft., and at a time of a flood, not of extraordinary height, flowing with a depth P equal to 2.5 ft., and having also an inclination of 2 ft. per mile. With the above relation of mean width to depth, we may, with all ordinary side-slopes, assume the transverse section to be rectangular; and therefore, with the above data, we shall have the hydraulic mean depth equal to 2.33 ft., and the mean velocity V equal to 2.75 ft. per sec.; consequently, the volume flowing down the river-course will be 481 cb. ft. per sec. If, then, we find a weir 4.5 ft. high constructed across the bed, and of such length, as, combined with the discharge through flood-sluices, or under mill-wheels at the site, shall give a depth of 1.5 ft. on the crest of the weir, the resulting depth p_0 at the section AXY, Fig. 75, will be 6 ft.; we may therefore proceed to calculate the length z_0 at which the depth of the back-water will be p_1 equal to 5.5 ft., or six inches less than at p_0. With an inclination of 2 ft. per mile, the value of sin i will be $\frac{1}{2640}$, or 0.00038. The velocity in z_0 being supposed uniform, and equal to that in the section AXY, Fig. 75, we find its value by the proportion,

Area at AXY : area at M : : vel^y in M : vel^y in AXY;

and thus 2.75 ft. being that at M, we have $v_0 = 1.146$ ft. per sec. The value of H_y, for the section at AXY, is 5.122. Substituting, therefore, all these numerical values in equation (10), we have

$$z_0 = 6 - 5.5 \left[\frac{64.4 \times 6 \qquad\qquad - 1.146^2 \times 2}{64.4 \times 6 \times .00038 - 1.646^2 \times .0064 \times (6 \div 5.122)}\right].$$

$$= 0.5 \times \frac{386.4 - 2.626}{0.1468 - 0.00975} = 0.5 \times \frac{383.77}{0.13705} = 1401 \text{ ft.}$$

Again, let us substitute numbers for the length z_4, we have then, taking the difference of depth in each length as 0.5 ft., $p_4 = 4$ ft., and $v_4 = 1.719$ ft. per sec., and thus,

$$z_4 = 4 - 3.5 \left[\frac{64.4 \times 4 \qquad\qquad - 1.719^2 \times 2}{64.4 \times 4 \times .00038 - 1.719^2 \times .0064 \times (4 \div 3.59)}\right].$$

$$= 0.5 \times \frac{251.69}{0.0768} = 1639 \text{ ft.}$$

The values of the several lengths, thus calculated, are as follows:—

z_0,	z_1,	z_2,	z_3,	z_4,	z_5,	z_6,
1401,	1434,	1477,	1563,	1639,	1870,	2482,

in all equal to 11865 ft., or 2 miles ¼, being ½ mile more than the hydrostatic amplitude, which is equal to 1¾ miles.

It must be remarked that the vertical rise of the water of the same flood is, even at well-constructed weirs, greater in the part of the course just below the site than in that immediately above. The fact being, that the velocity acquired by the water in descending the sloping surface of the weir is, generally, nearly all absorbed in eddies, and counter-currents, at the foot of the weir, from its entering the body of water below at a considerable angle with the horizon: there must, therefore, be a vertical elevation of the surface of the water below the weir, in order that it may have sufficient head to carry down the volume delivered over the weir; and this head will, in a great measure, depend on the form of the channel down stream. Thus, at Athlone weir, on the Shannon, it has been found that a flood rose on the lower sill of the lock contiguous to the weir, about 1 ft. 6 in. higher than it did at the same time on the upper sill. The longitudinal section of the so altered surface of the river must therefore have assumed a curved line down stream, gradually approximating to the line of the unaltered course, somewhat analogous to that curved line we have above considered as produced up stream by an impediment placed in the bed.

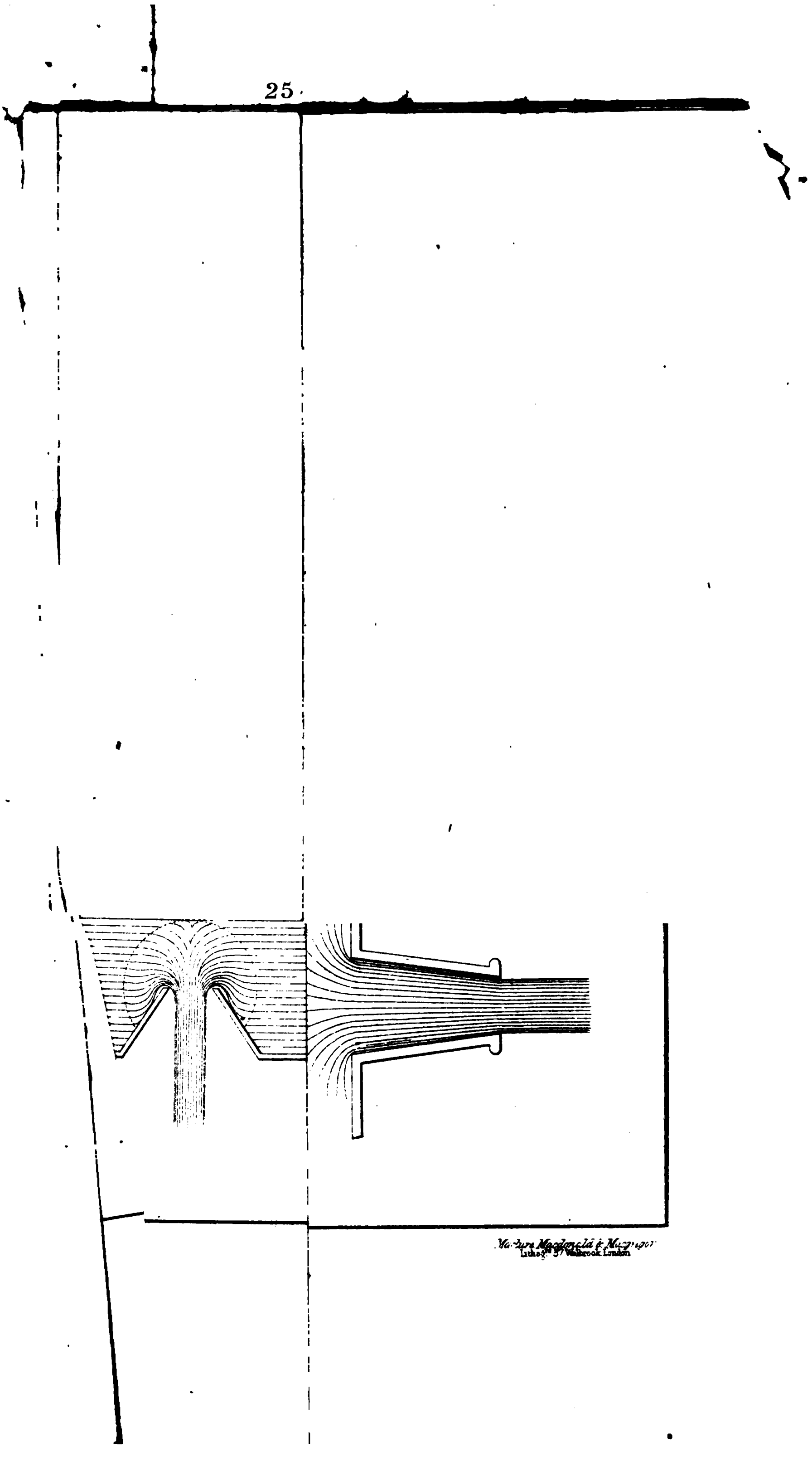
Now, if a new weir were erected below one already existing, and consequently raised the surface-line of the water in the reach above it; and, if from the proximity of those weirs, these two altered surfaces have not length, in ordinary floods, sufficient to merge independently into the surface-line of the intermediate part of the river uninfluenced by either, we should then, most probably, observe that the reactions of these two effects produced a new surface-line, very different from that given by the segments of each curve up to the point of intersection. The back-water on the upper weir might be found from this unsuspected cause to have been thrown back far beyond the common point of intersection, and probably assume a higher surface-line, and thus have given a reasonable ground of complaint to mill-owners at the upper weir.

A study of a few of the recorded examples of tidal surface of rivers will give a due impression of the effect of the resistances to the flow of water in river-courses.

In these it is observed that, when it is high-water on the bar at the river-mouth, the volume of water that has entered during flood-tide, instead of being in the form of a column extending up stream, with a horizontal line of surface, is found, on the contrary, to resemble a wedge with its surface inclined downwards from the river-mouth up stream, a form which can only arise from the resistance to the full ingress of the flood-tide caused by curves, and by the inequality of width and depth in different parts of the channel, and notably increased by lateral estuaries when such occur. Thus, in the River Tyne, which is not in any degree more unfavourably circumstanced than other rivers as to the form of its bed and shore, we find it is high-water at the bar at the river mouth when it is 2 ft. 6 in. below high-water at Newcastle-bridge, 9½ miles up, and dead low-water 15½ miles from the bar.

These inequalities, &c., in the river-bed exist in the upper part of the course also, and must produce a like effect on the surface of descending rain-floods, which, in passing downwards, may be said to resemble a tidal wave in some respects.

THE END.



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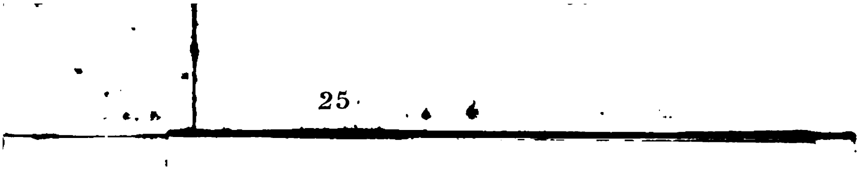

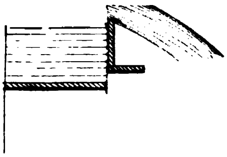

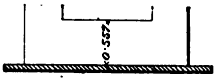

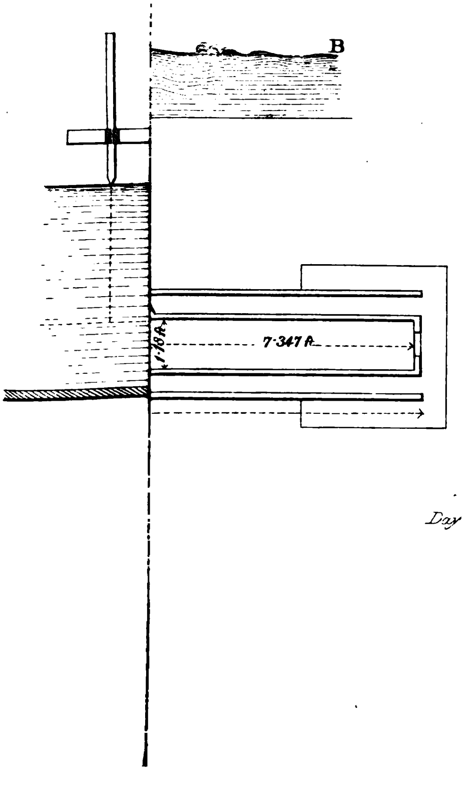

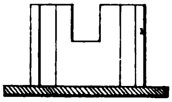
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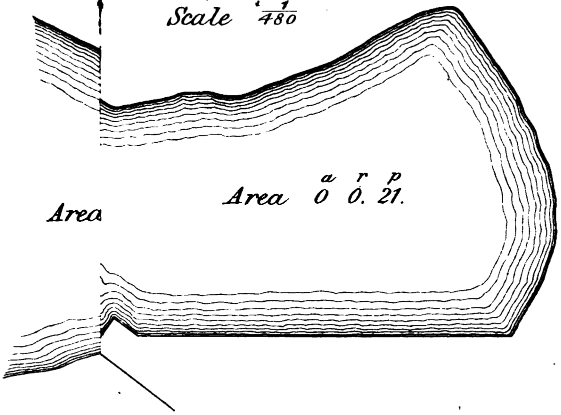
Day & Son, Lithʳˢ to the Queen

25

Scale $\frac{1}{480}$



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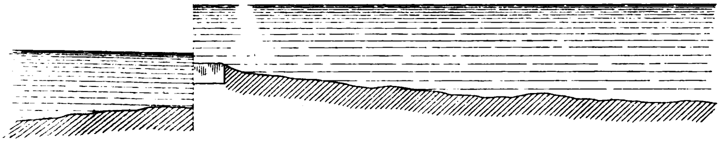

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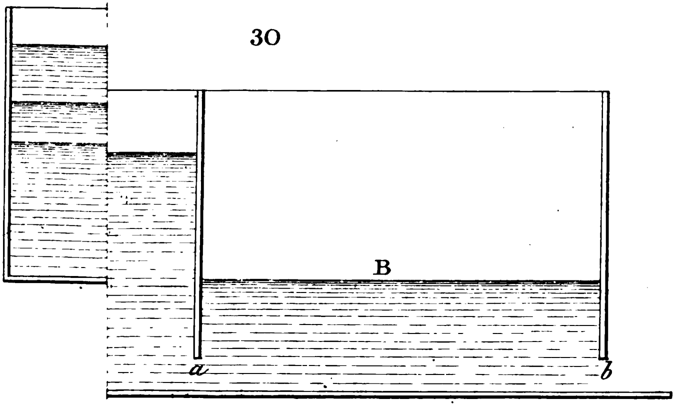

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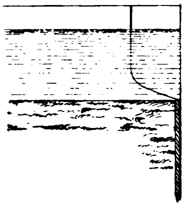

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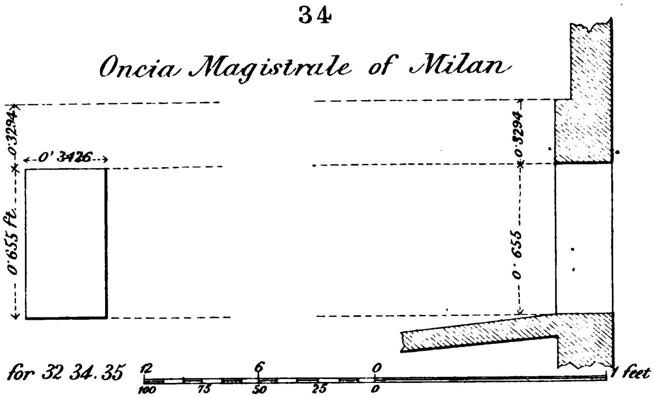
Oncia Magistrale of Milan



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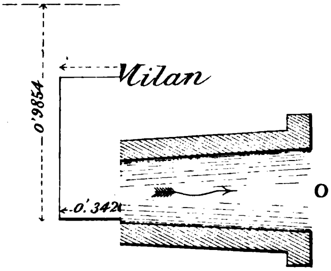

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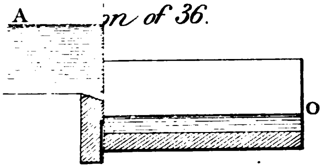

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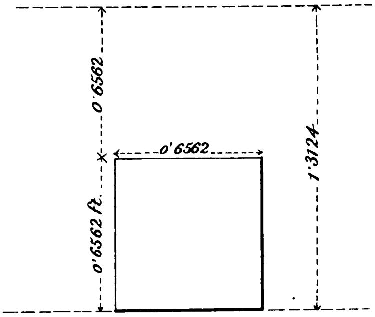

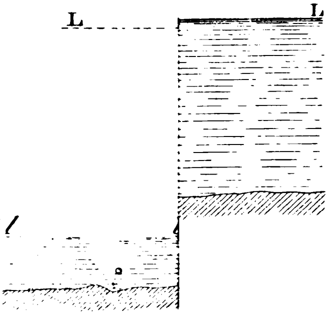

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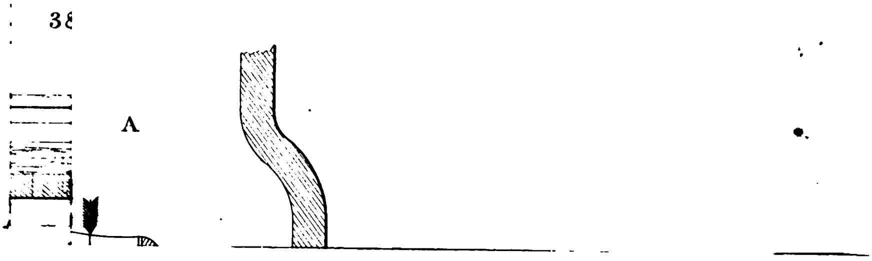
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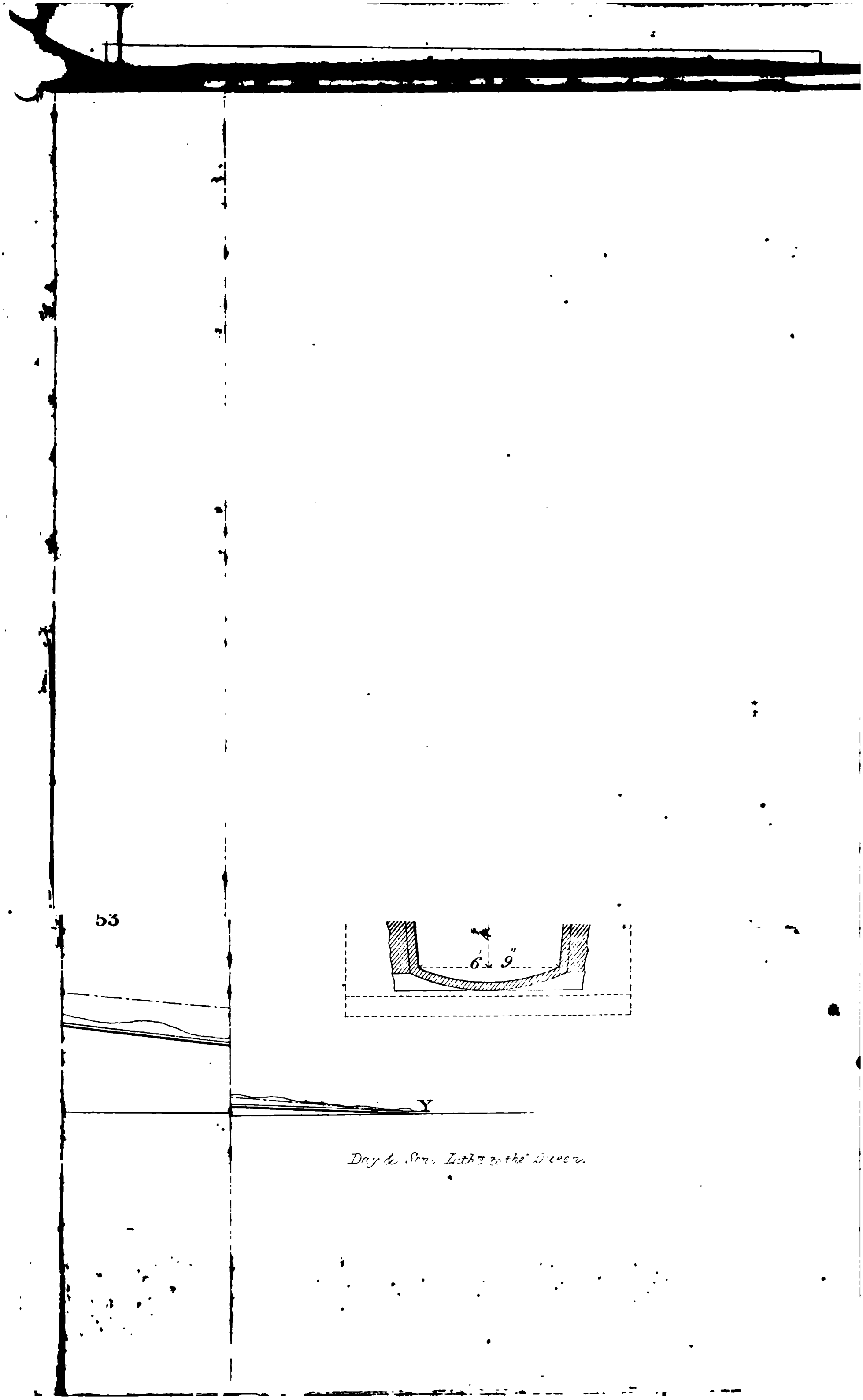

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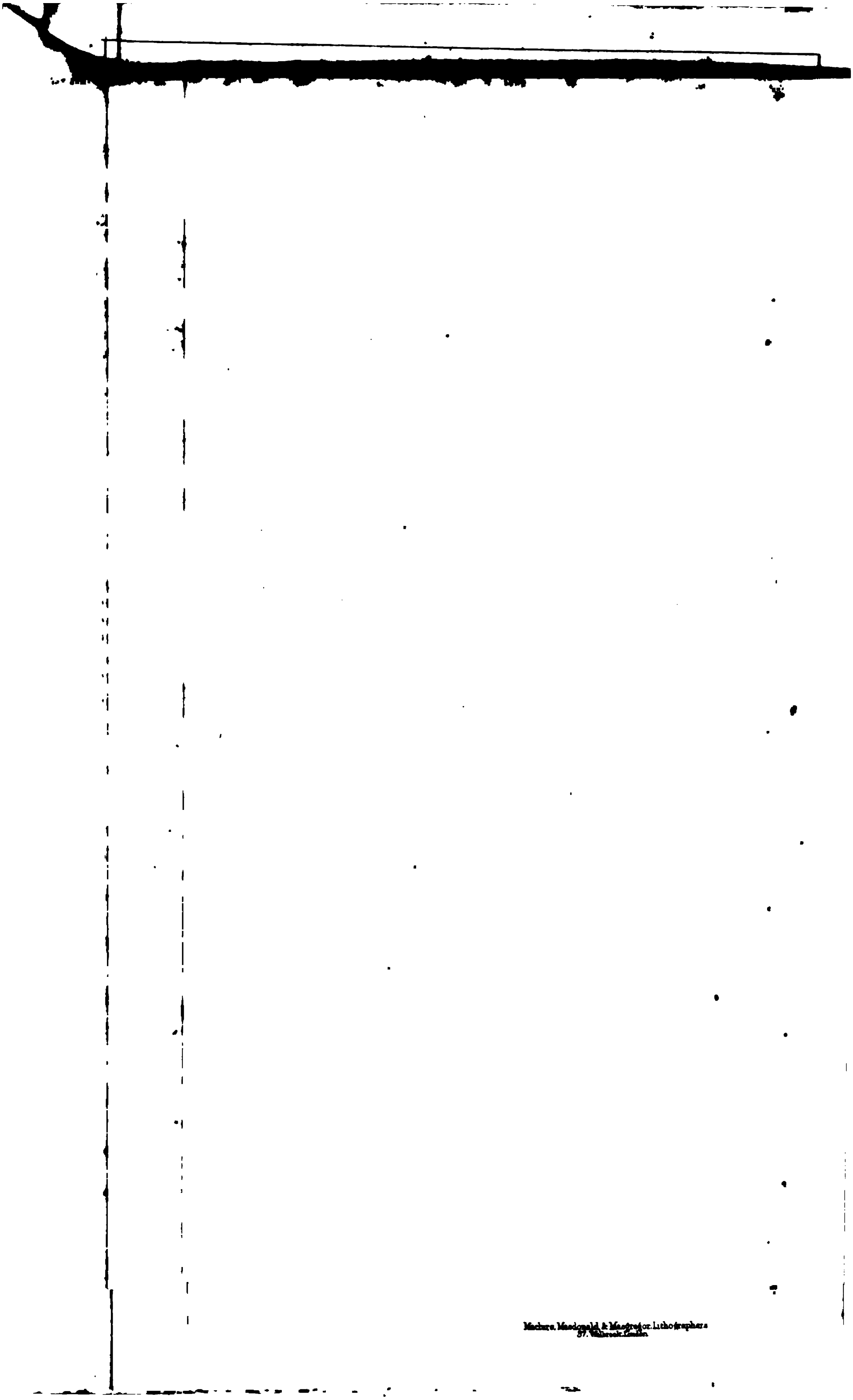

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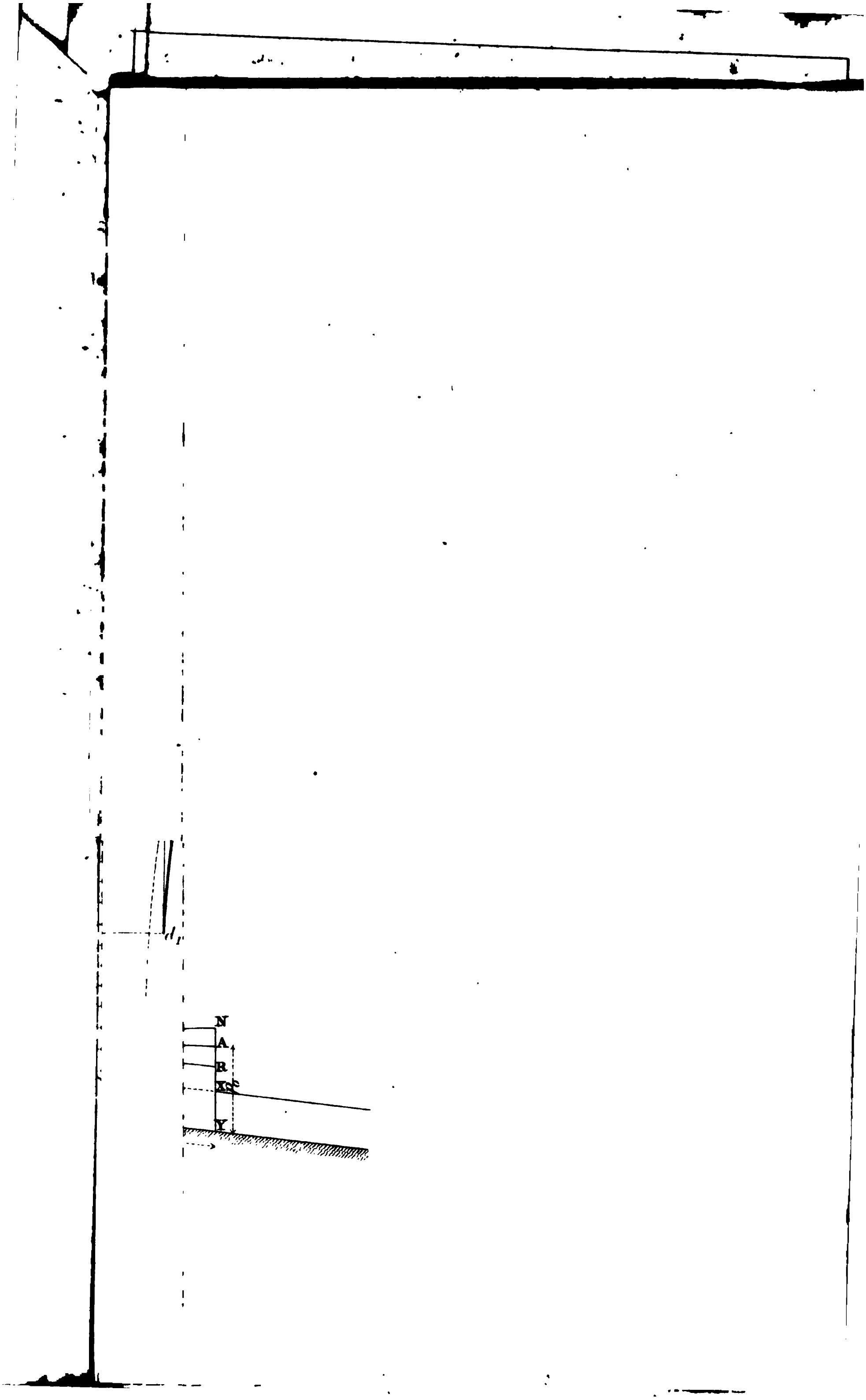
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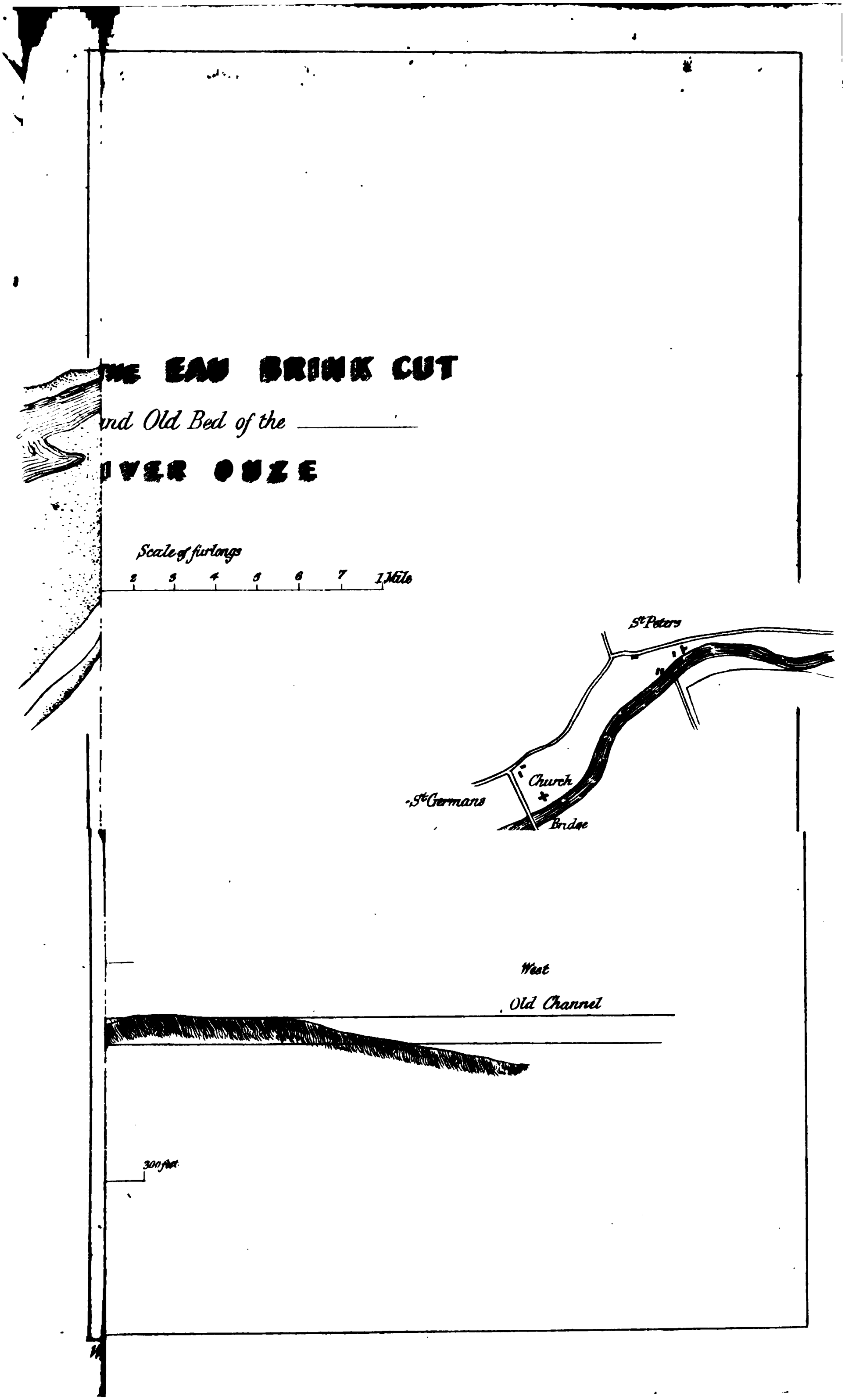
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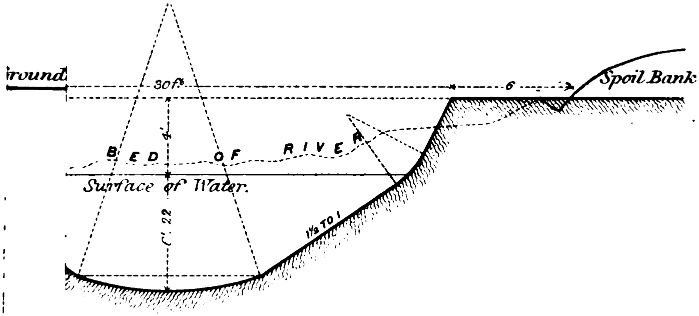

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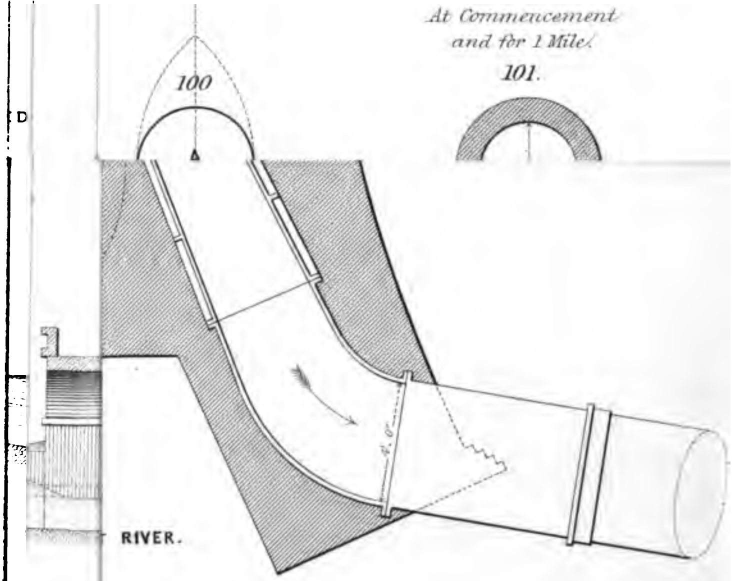

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